Multicarrier Equalization: Unification and Evaluation. Part I: Optimal Designs

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Abstract

Equalization is crucial in mitigating inter-carrier and inter-symbol interference. To ease equalization in a multicarrier system, a cyclic prefix (CP) is typically inserted between successive symbols. When the channel order exceeds the CP length, equalization can be accomplished via a time-domain equalizer (TEQ), which is a finite impulse response (FIR) filter. The TEQ is placed in cascade with the channel to produce an effective shortened impulse response. Alternatively, a bank of equalizers can be used to remove the interference tone-by-tone. This paper presents a unified treatment of optimal equalizer designs for multicarrier receivers. It is shown that almost all equalizer designs share a common mathematical framework that is based on the maximization of a product of generalized Rayleigh quotients. This framework is used to give an overview of existing designs, to apply a unified notation, and to present various common strategies to obtain a solution. Moreover, the unification emphasizes the differences between the methods, which enables a comparison of their advantages and disadvantages. In addition, an extensive literature survey is given.

Index Terms: Multicarrier, Channel Shortening, Time-domain Equalization, Optimal.

EDICS Designation: 3-TDSL, Telephone Networks and Digital Subscriber Loops

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I. Introduction to multicarrier equalization

During the last decades, extensive research has been done to provide broadband communication to and from the customer premises. To cope with the time dispersive transmission characteristics of wireline and wireless communications, multicarrier (MC) modulation offers a viable solution. In the 1960s, the first MC systems were conceived and implemented [1], [2], albeit only in analog form. In 1971, a widespread interest was created due to an all-digital implementation based on the cost-effective fast Fourier transform (FFT) algorithm [3]. Today, MC modulation is used in digital audio/video broadcasting [4], [5], in wireless local area networks such as IEEE 802.11a [6] and HIPERLAN2 [7], and in digital subscriber lines (DSL) [8], [9], [10], [11].

The multicarrier system model is shown in Fig. 1. The binary input data stream is split into N groups of bits, which are then passed through N ”constellation mappers [commonly quadrature amplitude modulation (QAM)]. The N complex-valued outputs are passed through an N-point inverse discrete Fourier transform (IDFT), implemented by the inverse FFT algorithm. The resulting finite-length time-domain signal is the symbol. After the signal passes through a physical channel, the receiver uses a DFT to recover the data within a bit error rate tolerance.

MC systems based on discrete multitone (DMT) modulation as defined in asymmetric and very high speed DSL (ADSL, VDSL) standards and orthogonal frequency division multiplexing

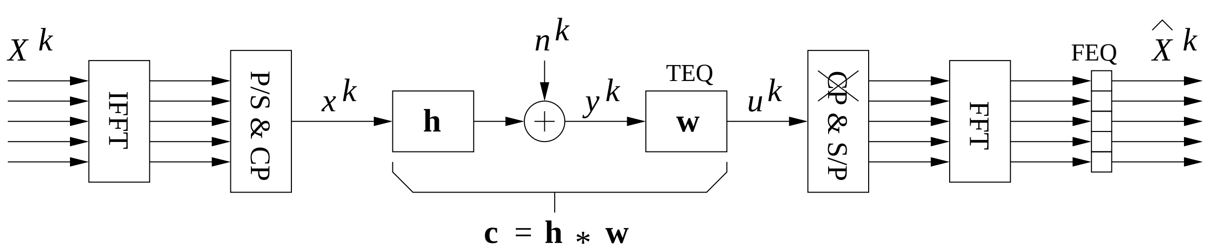

Fig. 1. Multicarrier system model. (I)FFT: (inverse) fast Fourier transform, P/S: parallel to serial, S/P: serial to parallel, CP: add cyclic prefix, and xCP: remove cyclic prefix.

(OFDM) as defined in IEEE 802.11a and HIPERLAN2 standards use an elegant equalization method. A cyclic prefix (CP), consisting of a copy of the last ν samples of each symbol, is prepended to the start of the symbol before transmission [12], [13]. If $\nu \geq L_h$, where L_h is the FIR channel order, the linear convolutive channel is converted to a circular one. The circulant convolution matrix is diagonalized by the IDFT and DFT matrices, so the transmitted data can be recovered by a bank of complex scalars, called a frequency-domain equalizer (FEQ). This channel order condition is often true for wireless OFDM; see [14] for a good overview. When $L_h > \nu$, which is e.g. the case for ADSL modems, the convolution is no longer circular, which results in inter-symbol and inter-carrier interference (ISI, ICI) [15]. To mitigate this interference, a time-domain equalizer (TEQ), which is an FIR filter, can be introduced before the FFT.

The goal of TEQ design is application-dependent: in a wireless scenario, bit-error rate minimization and fast adaptation to nonstationary environments are desired; whereas in ADSL, bitrate maximization in a quasi-stationary environment is targeted. This paper focuses on the DSL context, in which the ultimate performance measure is the achievable bit rate[1]. Most TEQ designs have been proposed in the DSL context. TEQ design has inspired many researchers because bitrate optimization leads to a highly non-linear optimization problem. Hence, simplified procedures are resorted to, which are primarily based on *time domain channel shortening*. Here, the TEQ is designed so that the convolution of the channel $\mathbf{h}$ (modeled as an FIR filter including transmit/receive front end filters and the physical transmission medium) and the TEQ $\mathbf{w}$ produces an overall impulse response with almost all of its energy concentrated in a length $\nu+1$ window.

The intriguing problems encountered in TEQ design are mainly due to the multicarrier demodulation using the FFT. Since the TEQ is before the FFT, all frequency bins are treated in a combined fashion. Moreover, the poor spectral containment of the demodulating FFT imposes a difficult interference structure and may lead to noise enhancement combined with "noise pick-up" from out-of-band noise [18]. Alternatively, one could consider a bank of equalizers, one per subcarrier. This approach is a generalization of the TEQ, which means that its performance should be as good as or better than the optimal TEQ. An even more general receiver structure exists where the cascade of the FFT and the equalizer filterbank is replaced by a new set of

[1] We will focus only on equalization issues without optimizing the power allocation of the different carriers. In reality, bitrate is truly optimized when equalization and power allocation are optimized jointly, which is a non-trivial problem [16], [17].

parallel filters [19], [20], [21]. The set of parallel filters act directly on the time domain samples to estimate the transmitted frequency domain symbols, without performing an FFT.

This paper presents an overview of the various equalizer designs. The goal of this two-part paper is to provide a unified mathematical framework and a unified notation for different equalizer designs, an extensive literature survey, and an objective evaluation in terms of performance and complexity. Although channel shortening could also be generalized to the multiple-input, multiple-output (MIMO) case in the wireless context [22], [23], [24], [25], [26] we will restrict ourselves to the single-input, single-output (SISO) transmission model.

The paper is organized as follows. The unified TEQ design framework is formulated in Section II. Single Rayleigh quotient designs are discussed in Sections III and IV, with a single filter or multiple filters, respectively. Designs with multiple Rayleigh quotients are discussed in Section V. Exceptions to the common formulation are treated in Section VI and conclusions are given in Section VII. Part II will discuss implementation issues (such as complexity reduction) and communications performance analysis through extensive simulations.

Throughout the paper, the following notation will be used:

- N is the (I)DFT size, ν is the prefix length, $s = N + \nu$ is the symbol size, N_u is the number of used tones, $\mathcal{S}$ is the set of used tones, i is the tone index, k is the DMT symbol index, n is the sample index, and Δ is the synchronization delay.
- $\mathcal{F}_N$ and $\mathcal{I}_N$ are the N-point DFT and IDFT matrices, respectively; $\mathbf{f}_i$ is the i-th DFT row.
- The transmitted (QAM) frequency domain symbol vector at time k is X^k; its i-th entry is X_i^k; vectors $\mathbf{x}^k$, $\mathbf{y}^k$, $\mathbf{n}^k$, and $\mathbf{u}^k$ contain the transmitted time domain samples, received samples (before the TEQ), additive noise samples, and TEQ output samples, respectively.
- $\mathbf{w}$, $\mathbf{h}$, and $\mathbf{c} = \mathbf{h} \star \mathbf{w}$ are vectors containing the TEQ, channel, and effective channel impulse responses of orders L_w, L_h, and L_c, respectively. $\star$ denotes linear convolution.
- $\mathbf{0}_{m \times n}$ is the all zero matrix of size $m \times n$; $\mathbf{I}_n$ is the identity matrix of size $n \times n$.
- $(\cdot)^T, (\cdot)^H, (\cdot)^*$ denote transpose, Hermitian, and complex conjugate respectively.
- $\mathcal{E}\{\cdot\}$ denotes statistical expectation.

II. COMMON FORMULATION

There are many ways of designing the DMT equalizer, depending on how the optimization problem is posed. However, almost all of the algorithms fit into the same formulation: the maximization of a generalized Rayleigh quotient or a product of generalized Rayleigh quotients. Consider the optimization problem

$$\boxed{\widehat{\mathbf{w}}^{opt} = \arg\max_{\widehat{\mathbf{w}}} \prod_{j=1}^{M} \frac{\widehat{\mathbf{w}}^T \mathbf{B}_j \widehat{\mathbf{w}}}{\widehat{\mathbf{w}}^T \mathbf{A}_j \widehat{\mathbf{w}}}} \tag{1}$$

In general, the solution to (1) is not well-understood when $M > 1$. However, for $M = 1$,

$$\boxed{\widehat{\mathbf{w}}^{opt} = \arg\max_{\widehat{\mathbf{w}}} \frac{\widehat{\mathbf{w}}^T \mathbf{B} \widehat{\mathbf{w}}}{\widehat{\mathbf{w}}^T \mathbf{A} \widehat{\mathbf{w}}}}, \tag{2}$$

the solution is the generalized eigenvector of the matrix pair $(\mathbf{B}, \mathbf{A})$ corresponding to the largest generalized eigenvalue [27]. Equivalently, the inverse of the ratio in (2) is minimized by the eigenvector of $(\mathbf{A}, \mathbf{B})$ corresponding to the smallest generalized eigenvalue. Most TEQ designs fall into the category of (2), although several have $M \gg 1$ as in (1). The vector $\widehat{\mathbf{w}}$ to be optimized is usually the TEQ taps, but it may also be e.g. the (shortened) target impulse response (TIR) [28], the per-tone equalizer taps [29], or half of the taps of a symmetric TEQ [30].

TEQ designs of the form of (2) include the Minimum Mean Squared Error (MMSE) design [28], [31], the Maximum Shortening SNR (MSSNR) design [32], the MSSNR design with a unit norm TEQ constraint (MSSNR-UNT) or a symmetric TEQ constraint (Sym-MSSNR) [33], the Minimum Inter-symbol Interference (Min-ISI) design [34], the Minimum Delay Spread (MDS) design [35], and the Carrier Nulling Algorithm (CNA) [36]. When a separate filter is designed for each tone, as in the per-tone equalizer (PTEQ) [29] or the TEQ filter bank (TEQFB) [37], [38], each filter can be designed by solving (2) separately for each tone.

The generalized eigenvector problem requires computation of the $\widehat{\mathbf{w}}$ that satisfies [27], [39]

$$\mathbf{B}\ \widehat{\mathbf{w}} = \lambda\ \mathbf{A}\ \widehat{\mathbf{w}}, \tag{3}$$

where $\widehat{\mathbf{w}}$ corresponds to the largest generalized eigenvalue λ. If $\mathbf{A}$ is invertible, the problem can be reduced to finding an eigenvector of $\mathbf{A}^{-1}\mathbf{B}$ [27]. When $\mathbf{A}$ is symmetric, another approach is to form the Cholesky decomposition $\mathbf{A} = \sqrt{\mathbf{A}}\sqrt{\mathbf{A}}^T$, and define $\hat{\mathbf{v}} = \sqrt{\mathbf{A}}^T\widehat{\mathbf{w}}$, as in [32]. Then

$$\hat{\mathbf{v}}^{opt} = \arg\max_{\hat{\mathbf{v}}} \frac{\hat{\mathbf{v}}^T \overbrace{\left(\sqrt{\mathbf{A}}^{-1}\ \mathbf{B}\ \sqrt{\mathbf{A}}^{-T}\right)}^{\mathbf{C}} \hat{\mathbf{v}}}{\hat{\mathbf{v}}^T \hat{\mathbf{v}}}. \tag{4}$$

The solution for $\hat{\mathbf{v}}$ is the eigenvector of $\mathbf{C}$ associated with the largest eigenvalue, and $\widehat{\mathbf{w}} = \sqrt{\mathbf{A}}^{-T}\hat{\mathbf{v}}$, assuming that $\mathbf{A}$ is invertible. If $\mathbf{A}$ is not invertible, then it has a non-zero null space, so the ratio is maximized (to infinity!) by choosing $\widehat{\mathbf{w}}$ to be a vector in the null space of $\mathbf{A}$.

In some cases, $\mathbf{A}$ or $\mathbf{B}$ is the identity matrix, in which case (3) reduces to a traditional eigenvalue problem. Examples include the computation of the MMSE target impulse response (TIR) [28], the MSSNR TEQ with a unit norm constraint on the TEQ [33], the MDS algorithm [35], and CNA [36]. There is a variety of all-purpose methods available for finding extreme eigenvectors, such as the power method [39] and conjugate gradient methods [40]. More specific iterative eigensolvers may be designed for specific problems, such as the MERRY (Multicarrier Equalization by Restoration of RedundancY) algorithm [41] and Nafie and Gatherer's method [42], which adaptively/iteratively compute the MSSNR TEQ.

The much more difficult case when $M > 1$ in (1) is not well-understood. There may be many solutions that are locally optimal but not necessarily globally optimal, so gradient-descent strategies only ensure convergence to a local optimum. Some TEQ algorithms of this form are the maximum geometric SNR (MGSNR) [43], Maximum Bit Rate (MBR) [34], Maximum Data Rate (MDR) [37], and Bitrate Maximum TEQ (BM-TEQ) [44], [45] methods. One approach is to compute several reasonable initial guesses, apply gradient descent (or a Newton-like algorithm) to each initialization, and then pick the best solution. This is not guaranteed to be optimal. The initial guesses can be made by computing the closed-form solutions for various $M = 1$ cases, such as the MSSNR TEQ or TEQs that optimize the bit rate on individual tones [37].

The motivation for introducing the common framework of (1) is to show how almost all TEQ designs require similar solution techniques; and to show how the designs differ in terms of the number of generalized Rayleigh quotients M, the choices of the matrices $\mathbf{A}_j$ and $\mathbf{B}_j$, and how $\mathbf{A}_j$ and $\mathbf{B}_j$ arise. This allows for increased understanding of how the different designs perform.

The next four sections deal with the cases $M = 1$ for a single filter, $M = 1$ for multiple filters, $M > 1$ for a single filter, and exceptions to the rule, respectively. The various designs will be explained in detail, the similarities and differences will be expounded upon, and the existing knowledge of each method will be summarized.

III. Single quotient cases

Researchers have been investigating TEQ design for more than a decade now, and this research has lead to an overwhelming number of publications. Many methods can be reformulated as the maximization of a generalized Rayleigh quotient, as in (2). This section reviews the literature concerning these designs, and discusses the advantages and disadvantages of each method.

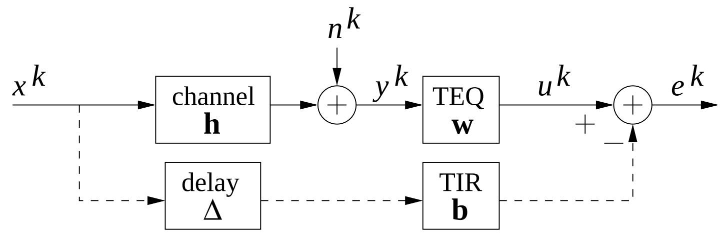

Fig. 2. Block diagram used for MMSE channel shortening.

A. *Minimum mean square error (MMSE)*

In the seventies, Falconer and Magee [28] introduced an MMSE method to shorten the impulse response of a communication channel for maximum likelihood (ML) receivers. Their objective was to design a filter prior to the Viterbi algorithm, which is frequently used for ML data sequence estimation. This pre-filtering attempts to reduce the memory of the overall channel, resulting in an exponential decrease in computational complexity of the Viterbi algorithm.

In the early nineties, Chow and Cioffi [46] extended the MMSE channel shortening problem to time domain equalization in multicarrier systems. In [46], a finite and an infinite length TEQ are computed to shorten the channel impulse response (CIR) to a $\nu+1$ tap target impulse response (TIR). In this paper, we will focus on the finite length case.

The MMSE TEQ design is depicted in Fig. 2. Define $\mathbf{x}^k$ as a vector of $\nu+1$ transmitted samples and $\mathbf{y}^k$ a vector of L_w+1 received samples at time k. The transmitted signal $\mathbf{x}^k$ is passed through the CIR $\mathbf{h}$ and is equalized by the TEQ $\mathbf{w}$. The equalizer output is compared with the input signal filtered by the TIR $\mathbf{b}$ and delayed with Δ. The difference e^k is then minimized in the mean square sense with respect to $\mathbf{w}$ and $\mathbf{b}$, i.e. the cost function can be expressed as

$$J(\mathbf{w},\mathbf{b}) = \mathcal{E}\{(e^k)^2\} = \mathcal{E}\{(\mathbf{w}^T\mathbf{y}^k - \mathbf{b}^T\mathbf{x}^{k-\Delta})^2\}, \tag{5}$$

$$= \mathbf{w}^T\mathbf{R}_y\mathbf{w} + \mathbf{b}^T\mathbf{R}_x\mathbf{b} - 2\mathbf{b}^T\mathbf{R}_{yx}(\Delta)^T\mathbf{w}, \tag{6}$$

where $\mathbf{R}_x = \mathcal{E}\{\mathbf{x}^k(\mathbf{x}^k)^T\}$, $\mathbf{R}_y = \mathcal{E}\{\mathbf{y}^k(\mathbf{y}^k)^T\}$, and $\mathbf{R}_{yx}(\Delta) = \mathcal{E}\{\mathbf{y}^k(\mathbf{x}^{k-\Delta})^T\}$, which is a function of the delay parameter Δ. Using the well-known *orthogonality principle* [47], which states that the optimal error sequence is uncorrelated to the input data, we obtain

$$\mathbf{w} = \mathbf{R}_y^{-1}\mathbf{R}_{yx}\mathbf{b}, \tag{7}$$

which allows reformulation of (6) as a function of $\mathbf{w}$ or $\mathbf{b}$ alone [28].

The trivial all-zero solution can be avoided by adding a constraint on the TEQ or TIR [8], [31], [46], [48], [49], [50]. The MMSE optimization problem with various constraints can be cast into the general problem formulation of (2) with different $\mathbf{A}$ and $\mathbf{B}$ matrices for the different constraints. Commonly used constraints are as follows:

1) unit-norm constraint on the TEQ, i.e. $\mathbf{w}^T\mathbf{w} = 1$:

$$\mathbf{A} = \mathbf{R}_y - \mathbf{R}_{yx}\mathbf{R}_x^{-1}\mathbf{R}_{xy}, \quad (8)$$

$$\mathbf{B} = \mathbf{I}_{L_w+1}. \quad (9)$$

As discussed in Section II, the optimal TEQ is the eigenvector corresponding to the smallest eigenvalue of $\mathbf{A}$. In [18], [49], [50], the unsatisfactory performance of this constrained design was reported. Under this constraint the TEQ typically boosts 'out of band' noise, which leaks into the band of interest due to the poor spectral containment of the demodulating DFT [18]. In order to concentrate the TEQ energy into the desired passband region, virtual (i.e. mathematical) noise can be injected into the stopband, using a modified $\mathbf{A}$ matrix:

$$\mathbf{A} = (\mathbf{R}_y + \mu\mathbf{D}) - \mathbf{R}_{yx}\mathbf{R}_x^{-1}\mathbf{R}_{xy}. \quad (10)$$

Here, μ controls the virtual noise level. The TEQ tries to suppress the virtual noise, thereby lowering the undesired noise enhancement [18].

2) unit-energy constraints, i.e. $\mathbf{w}^T\mathbf{R}_y\mathbf{w} = 1$, or $\mathbf{b}^T\mathbf{R}_x\mathbf{b} = 1$, or $\mathbf{w}^T\mathbf{R}_y\mathbf{w} = 1$ & $\mathbf{b}^T\mathbf{R}_x\mathbf{b} = 1$:

$$\mathbf{A} = \mathbf{R}_y - \mathbf{R}_{yx}\mathbf{R}_x^{-1}\mathbf{R}_{xy}, \quad (11)$$

$$\mathbf{B} = \mathbf{R}_y. \quad (12)$$

These different unit-energy constraints remarkably lead to the same TEQ coefficients, up to a scaling factor, which can be incorporated into the one-tap FEQs [49].

3) unit-norm constraint on the TIR, i.e. $\mathbf{b}^T\mathbf{b} = 1$. If X is white, then $\mathbf{R}_x$ is identity, so $\mathbf{b}^T\mathbf{R}_x\mathbf{b} = \mathbf{b}^T\mathbf{b}$ and the previous case with $\mathbf{R}_x = \mathbf{I}_{\nu+1}$ yields the optimal TEQ. Whether X is white or not, (2) can be reformulated with $\widehat{\mathbf{w}} = \mathbf{b}$ and [28]

$$\mathbf{A} = \mathbf{R}_x - \mathbf{R}_{xy}\mathbf{R}_y^{-1}\mathbf{R}_{yx}, \quad (13)$$

$$\mathbf{B} = \mathbf{I}_{\nu+1}. \quad (14)$$

After calculating the solution for $\mathbf{b}$, the TEQ coefficients can be obtained using (7).

4) unit-tap constraint on the TEQ, i.e. $\mathbf{e}_j^T \mathbf{w} = \pm 1$, where $\mathbf{e}_j$ is the elementary vector with element one in the jth position:

$$\mathbf{A} = \mathbf{R}_y - \mathbf{R}_{yx}\mathbf{R}_x^{-1}\mathbf{R}_{xy}, \tag{15}$$

$$\mathbf{B} = \mathbf{e}_j \mathbf{e}_j^T. \tag{16}$$

The first TEQ algorithm proposed for DMT transmission [51] also falls into this category. During modem initialization, an IIR channel model (with ν zeros and L_w poles) is derived,

$$H(z) = \frac{B(z)}{A(z)} = \frac{z^{-\Delta} \sum_{l=0}^{\nu} b_l z^{-l}}{1 + \sum_{l=1}^{L_w} a_l z^{-l}}, \tag{17}$$

where z is the $\mathcal{Z}$-transform variable. Based on this model, the receiver then sets its TEQ taps to a_l, for $l = 0, \ldots, L_w$, such that the effective channel corresponds to the numerator of $H(z)$, which is confined in extent to be CP length plus one taps. One can easily verify that a classical linear prediction method to estimate the numerator and denominator coefficients of $H(z)$ is equivalent to computing an MMSE TEQ with $\mathbf{e}_1^T \mathbf{w} = 1$. This idea was extended to a multiple-input-single-output filter bank at the receiver in [52].

5) unit-tap constraint on the TIR, i.e. $\mathbf{e}_j^T \mathbf{b} = \pm 1$. The optimal TIR uses (2) with $\widehat{\mathbf{w}} = \mathbf{b}$ and

$$\mathbf{A} = \mathbf{R}_x - \mathbf{R}_{xy}\mathbf{R}_y^{-1}\mathbf{R}_{yx}, \tag{18}$$

$$\mathbf{B} = \mathbf{e}_j \mathbf{e}_j^T. \tag{19}$$

After calculating the solution for $\mathbf{b}$, the TEQ coefficients can be obtained using (7).

MMSE TEQ design has been extensively studied in the open research literature. Chow and Cioffi's basic results of [46] were further explored in [31] with emphasis on unit-tap and unit-norm constraints on the TIR. The authors of [31] have shown that a unit-norm constrained TIR results in a lower MMSE than a unit-tap constrained TIR. Given this fact, they conclude that the unit-norm constraint solution results in better performance. However, the MMSE performance measure is not directly related to bit rate maximization [34], [43] hence it is difficult to predict which constraint is preferable. In an attempt to improve the performance of the MMSE shortening algorithm, Van Acker *et al.* modified the cost function with frequency weighting to disregard unused frequency bins [50], [53], [54]. Although the authors report some improvement in bit rate,

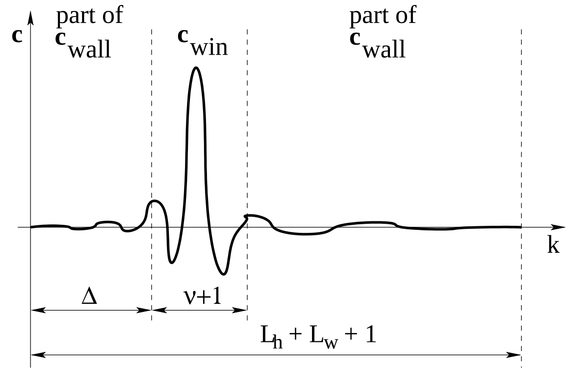

Fig. 3. The “window” and “wall” parts of the effective channel.

this approach still does not maximize achievable bit rate. Moreover, the bit rate is a non-smooth function of Δ, and thus optimizing the delay parameter requires a global search [29].

B. Maximum shortening SNR (MSSNR)

In 1996, Melsa, Younce, and Rohrs proposed the maximum shortening signal-to-noise ratio (MSSNR) method [32]. The MSSNR approach is based solely on shortening the CIR. Define

$$\mathbf{H}_{wall,1} = \begin{bmatrix} h_0 & 0 & \dots & 0 \\ \vdots & \ddots & & \vdots \\ h_{\Delta-1} & \dots & & h_{\Delta-L_w-1} \end{bmatrix}, \tag{20}$$

$$\mathbf{H}_{win} = \begin{bmatrix} h_\Delta & \dots & h_{\Delta-L_w} \\ \vdots & \ddots & \vdots \\ h_{\Delta+\nu} & \dots & h_{\Delta+\nu-L_w} \end{bmatrix} \tag{21}$$

$$\mathbf{H}_{wall,2} = \begin{bmatrix} h_{\Delta+\nu+1} & \dots & & h_{\Delta+\nu-L_w+1} \\ \vdots & \ddots & & \vdots \\ 0 & \dots & 0 & h_{L_h} \end{bmatrix}, \tag{22}$$

$$\mathbf{H}_{wall} = [\mathbf{H}_{wall,1}^T \;\; \mathbf{H}_{wall,2}^T]^T. \tag{23}$$

Recall that the effective channel is $\mathbf{c} = \mathbf{h} \star \mathbf{w}$. The MSSNR technique [32] attempts to *minimize* the energy *outside* a window of $\nu + 1$ consecutive samples of $\mathbf{c}$ (called the ‘wall’), while constraining the energy in the desired window of $\mathbf{c}$ to one, as shown in Fig. 3. The expressions for the energy outside and inside the window can be written as

$$\mathbf{c}_{wall}^T \mathbf{c}_{wall} = \mathbf{w}^T \mathbf{H}_{wall}^T \mathbf{H}_{wall} \mathbf{w}, \quad (24)$$

$$\mathbf{c}_{win}^T \mathbf{c}_{win} = \mathbf{w}^T \mathbf{H}_{win}^T \mathbf{H}_{win} \mathbf{w}, \quad (25)$$

respectively. The final TEQ coefficients are found as

$$\min_{\mathbf{w}} \mathbf{w}^T \mathbf{H}_{wall}^T \mathbf{H}_{wall} \mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^T \mathbf{H}_{win}^T \mathbf{H}_{win} \mathbf{w} = 1. \quad (26)$$

When $\mathbf{H}_{win}^T \mathbf{H}_{win}$ has a non-empty null space, i.e. when $\mathbf{H}_{win}$ has more columns than rows ($L_w > \nu$), solving (26) becomes rather complicated [32].

Alternatively, one can *maximize* the windowed energy, while constraining the wall energy, as suggested in [55] and later in [56]. Since $\mathbf{H}_{wall}^T \mathbf{H}_{wall}$ is always positive definite when $L_h \geq \nu+1$, the latter approach is preferred and reduces to solving (2) with

$$\mathbf{A} = \mathbf{H}_{wall}^T \mathbf{H}_{wall}, \quad (27)$$

$$\mathbf{B} = \mathbf{H}_{win}^T \mathbf{H}_{win}. \quad (28)$$

Maximizing window to wall energy provides the same TEQ as maximizing overall channel energy to wall energy [11], [57], [58]. The minimum inter-block interference (Min-IBI) method [59] is an MSSNR variant that weights the ISI terms linearly with their distance from the window.

The MSSNR approach tacitly assumes that the input signal is white. In the absence of noise, the MSSNR approach is equivalent to the MMSE design for a white input signal [60]. The MSSNR TEQ ignores noise, so it may be referred to as a zero-forcing (ZF) design. The MSSNR method can be extended to the noisy case by adding a noise correlation matrix to $\mathbf{A}$, i.e. [33]

$$\mathbf{A} = \mathbf{H}_{wall}^T \mathbf{H}_{wall} + \mathbf{R}_n, \quad (29)$$

$$\mathbf{B} = \mathbf{H}_{win}^T \mathbf{H}_{win}. \quad (30)$$

The *infinite length* MSSNR TEQ is always symmetric or skew-symmetric [30], [33], [61]. Oddly enough, the *finite* length MSSNR TEQ is almost always symmetric and not skew-symmetric. Thus, design complexity can be dramatically reduced by forcing a perfectly symmetric TEQ [30], [33] by rewriting $\mathbf{w}^T \mathbf{A} \mathbf{w}$ (with $\mathbf{A}$ as in (27) or (29)) as

$$\underbrace{\left[\widetilde{\mathbf{w}}^T, \quad \widetilde{\mathbf{w}}^T \mathbf{J}\right]}_{\mathbf{w}^T} \underbrace{\begin{bmatrix} \mathbf{A}_{11}, & \mathbf{A}_{12} \\ \mathbf{A}_{21}, & \mathbf{A}_{22} \end{bmatrix}}_{\text{MSSNR } \mathbf{A}} \underbrace{\begin{bmatrix} \widetilde{\mathbf{w}} \\ \mathbf{J}\widetilde{\mathbf{w}} \end{bmatrix}}_{\mathbf{w}} = \widetilde{\mathbf{w}}^T \underbrace{\left[\mathbf{A}_{11} + \mathbf{J}\mathbf{A}_{21} + \mathbf{A}_{12}\mathbf{J} + \mathbf{J}\mathbf{A}_{22}\mathbf{J}\right]}_{\text{Sym}-\text{MSSNR } \mathbf{A}} \widetilde{\mathbf{w}}, \quad (31)$$

where $\mathbf{J}$ is the square matrix with ones on the anti-diagonal, and $\widetilde{\mathbf{w}}$ is half the size of $\mathbf{w}$. A similar redefinition holds for $\mathbf{B}$. The desired symmetric TEQ is obtained via (2) with $\widehat{\mathbf{w}} = \widetilde{\mathbf{w}}$ and the $\mathbf{A}$ and $\mathbf{B}$ matrices redefined as

$$\mathbf{A} = \mathbf{A}_{11} + \mathbf{J}\mathbf{A}_{21} + \mathbf{A}_{12}\mathbf{J} + \mathbf{J}\mathbf{A}_{22}\mathbf{J}, \tag{32}$$

$$\mathbf{B} = \mathbf{B}_{11} + \mathbf{J}\mathbf{B}_{21} + \mathbf{B}_{12}\mathbf{J} + \mathbf{J}\mathbf{B}_{22}\mathbf{J}. \tag{33}$$

In [33], [62], it was reported that symmetric MSSNR TEQs have a comparable performance with respect to the MSSNR design of [32], with reduced computational complexity.

C. Multi-carrier equalization by restoration of redundancy (MERRY)

In [41], one of the few *blind* channel-shortening algorithms was presented. This method, called MERRY, exploits the CP redundancy to force the last sample in the equalized CP to be equal to the last sample in the equalized symbol. The cost function that reflects this principle is

$$J(\mathbf{w}) = \mathcal{E}\left\{\left|\mathbf{u}^k\left[\nu + \Delta\right] - \mathbf{u}^k\left[\nu + N + \Delta\right]\right|^2\right\}, \tag{34}$$

where $\mathbf{u}^k[j]$ denotes the jth element of the symbol vector after the TEQ. From (34), it follows that MERRY attempts to produce a windowed effective channel of ν taps instead of $\nu+1$. Under the assumption of a white input signal, Martin *et al.* showed that (34) produces a TEQ which minimizes the wall portion of the effective channel (like the MSSNR design) under a unit norm constraint $\mathbf{w}^T\mathbf{w} = 1$ (or a unit energy constraint [58]) while limiting the noise gain [41]. The MERRY TEQ design can be reformulated as a single generalized Rayleigh quotient optimization as in (2) with

$$\mathbf{A} = \tilde{\mathbf{H}}_{wall}^T\tilde{\mathbf{H}}_{wall} + \mathbf{R}_n, \tag{35}$$

$$\mathbf{B} = \mathbf{I}_{L_w+1}, \tag{36}$$

where $\tilde{\mathbf{H}}_{wall}$ contains one extra row compared to (23).

D. Minimum intersymbol interference (Min-ISI)

A generalization of the MSSNR method was given in [34] and [63], referred to as the minimum ISI (min-ISI) method. Arslan, Evans, and Kiaei [34] define the sub-channel SNR as

$$\mathrm{SNR}_i = \frac{S_{x,i}\left|C_{signal,i}\right|^2}{S_{n,i}\left|C_{noise,i}\right|^2 + S_{x,i}\left|C_{ISI,i}\right|^2}, \tag{37}$$

where $S_{x,i}$, $S_{n,i}$, $C_{signal,i}$, $C_{noise,i}$ and $C_{ISI,i}$ are the transmitted signal power, channel noise power, signal path gain, noise path gain, and the ISI path gain in the ith sub-channel, i.e.

$$C_{signal,i} = \mathbf{f}_i \, \mathrm{diag}\,(\mathbf{g}) \, \mathbf{H} \, \mathbf{w}, \tag{38}$$

$$C_{ISI,i} = \mathbf{f}_i \, (\mathbf{I}_N - \mathrm{diag}\,(\mathbf{g})) \, \mathbf{H} \, \mathbf{w} \stackrel{\triangle}{=} \mathbf{f}_i \, \mathbf{D} \, \mathbf{H} \, \mathbf{w}, \tag{39}$$

$$C_{noise,i} = \mathbf{f}_i \, [\mathbf{w}^T, \quad \mathbf{0}_{1\times(N-L_w-1)}]^T. \tag{40}$$

Here, the $N \times 1$ vector $\mathbf{g}$ and the $N \times (L_w + 1)$ convolution matrix $\mathbf{H}$ are given by

$$\mathbf{g}[n] = \begin{cases} 1, & \Delta \leq n \leq \Delta + \nu \\ 0, & \text{otherwise} \end{cases} \tag{41}$$

$$\mathbf{H} = [\mathbf{H}_{wall,1}^T, \quad \mathbf{H}_{win}^T, \quad \mathbf{H}_{wall,2}^T]^T, \tag{42}$$

which makes use of (20), (21), and (22). According to [63], the matched filter bound on the SNR is obtained when each sub-carrier ISI term of (37) is forced to zero. As a consequence, they propose to minimize a sum of the sub-channel ISI terms. The TEQ design satisfies (2) with

$$\mathbf{A} = \mathbf{H}^T \mathbf{D}^T \left(\sum_{i \in \mathcal{S}} \mathbf{f}_i^H S_{x,i} \mathbf{f}_i \right) \mathbf{D}\mathbf{H}, \tag{43}$$

$$\mathbf{B} = \mathbf{H}_{win}^T \mathbf{H}_{win}, \tag{44}$$

where $\mathcal{S}$ denotes the set of used tones and matrix $\mathbf{B}$ acts as a constraint to prevent the trivial all-zero TEQ solution by maintaining $\mathbf{w}^T\mathbf{B}\mathbf{w} = 1$. In [34], channel noise coloration was taken into account by modifying (43) and (44) into

$$\mathbf{A} = \mathbf{H}^T \mathbf{D}^T \left(\sum_{i \in \mathcal{S}} \mathbf{f}_i^H \frac{S_{x,i}}{S_{n,i}} \mathbf{f}_i \right) \mathbf{D}\mathbf{H}, \tag{45}$$

$$\mathbf{B} = \mathbf{H}_{win}^T \mathbf{H}_{win}. \tag{46}$$

The conventional subchannel SNR ratio of $\frac{S_{x,i}}{S_{n,i}}$ in (45) forces the ISI to be placed in subchannels with low conventional subchannel SNR. Comparing (27) and (28) to (45) and (46), the residual ISI-noise is now shaped in the frequency domain. The min-ISI method of [34] can be considered as a generalization of the MSSNR method [32], since both methods would be equivalent if the SNR were constant over all sub-channels and if all sub-channels were used.

The dual-path TEQ [64] makes use of the Min-ISI design. One TEQ is designed for all of the tones, and then a second TEQ is designed in parallel using the Min-ISI method for a small subset of tones. The subset is chosen as the low-frequency tones which are expected to have a high bit rate.

The min-ISI method resulted from applying a simplification to the Maximum Bit Rate method (to be discussed in Section V-B) to make the approach tractable. However, it is still suboptimal in terms of bit rate performance. As the demodulating DFT length is finite, sub-carriers are not perfectly orthogonal, which results in inter-carrier interference (ICI). The ICI-noise component is neglected in (37). In addition, the signal path gain of (38) is an approximation. In practice, the head and the tail of the effective channel will contribute to the useful signal component [15].

E. Minimum delay spread (MDS)

The taps of $\mathbf{c}$ exceeding the CP length cause ISI and ICI, but the interference levels depend on the taps' distances to the prefix and their energy [35]. Therefore, Schur and Speidel [35] propose to minimize the square of the delay spread of $\mathbf{c}$, where the delay spread is given by

$$D = \sqrt{\frac{1}{E}\sum_{n=0}^{L_c}(n-\bar{n})^2\,|\mathbf{c}[n]|^2}. \tag{47}$$

Here, $E = \mathbf{c}^T\mathbf{c}$, and $\bar{n}$ is a user-defined "center of mass." This results in (2) with

$$\mathbf{A} = \mathbf{H}^T\mathbf{Q}\mathbf{H}, \tag{48}$$

$$\mathbf{B} = \mathbf{H}^T\mathbf{H}, \tag{49}$$

where $\mathbf{Q} = \mathrm{diag}\{[(0-\bar{n})^2, \;\; \ldots, \;\; (L_w + L_h - \bar{n})^2]\}$ is a diagonal weighting matrix.

Since the MDS TEQ does not exploit the cyclic prefix redundancy, it attempts to shorten the effective channel to a single spike. Since MDS TEQ design is quite similar to MSSNR TEQ design, except for a quadratic instead of a wall penalty function [65], the advantages and drawbacks mentioned in Section III-B also apply here.

F. Carrier nulling algorithm (CNA)

In a typical DMT/OFDM system, some frequency bins transmit only zeros, the so-called null-carriers. In [36], the authors propose a *blind* method to combat channel dispersion based on the minimization of the average DFT-output energy of the null carriers. The TEQ can be designed to force the received symbols on the null-carriers to zero by minimizing the cost function

$$J = \sum_{i \in \overline{\mathcal{S}}} \mathcal{E}\left\{|U_i^k|^2\right\}, \tag{50}$$

$$= \mathbf{w}^T(\mathbf{P}+\mathbf{Q})\mathbf{w}, \tag{51}$$

where $\overline{\mathcal{S}}$ represents the set of null-carriers, U_i^k is the DFT output on tone i, and $\mathbf{P}$ and $\mathbf{Q}$ denote signal and noise dependent matrices respectively (see [36] for complete definitions). The constraint $\mathbf{w}^T\mathbf{w} = 1$, is used to avoid the all-zero solution. This TEQ design can be cast into the problem formulation of (2) with

$$\mathbf{A} = \mathbf{P}+\mathbf{Q}, \tag{52}$$

$$\mathbf{B} = \mathbf{I}_{L_w+1}. \tag{53}$$

Although the authors presented a low-complexity, adaptive minimization procedure for (50) and the algorithm is blind, the CNA criterion only considers unused carriers without paying attention to the carriers of interest. This will not necessarily lead to channel shortening, nor equalization of the useful carriers. Specifically, de Courville *et al.* claim that their algorithm leads to shortening to a single spike [36] rather than to a window, though Romano and Barbarossa state that a MSSNR solution can be achieved by frequency-hopping the null tones [66].

IV. MULTIPLE FILTERS, EACH WITH A SINGLE QUOTIENT

In an ADSL or VDSL system, the TEQ design should involve maximizing the bit rate of the system. Traditionally, this has not been the case (see Section III) as most of the TEQ methods optimize a cost function that is not directly related to the bit rate equation defined by the standards. Notable exceptions are discussed in this section and the next section. Higher bit rate means that higher bandwidth content (more services) can be offered through a single ADSL or VDSL line. Alternatively, the ADSL or VDSL system margin can be improved, given a constant bit rate; or given a fixed bit rate and margin, the reach of a DMT system can be extended.

This section focuses on alternative equalizer designs that maximize the DMT system bit rate. Whereas in the previous section a single time-domain equalizer was designed to equalize all frequency bins together in some optimal way, the idea here is to tailor an equalizer design to each subchannel: each data-carrying subchannel receives an equalizer, which is designed to maximize the bit rate on that subchannel. By extension, if every subchannel carries the maximum number of bits, then the bit rate of the DMT system is also maximized. In terms of the common formulation given by (1), this means that a single generalized Rayleigh quotient ($M = 1$) is maximized per subchannel, which results in a separate solution for each subcarrier. This idea was originally presented in [29] as a "per-tone equalization" (PTEQ) architecture. An alternative formulation, called the "Time-Domain Equalizer Filter Bank" (TEQFB), is given in [37], [38].

A. Per-tone equalization

The PTEQ scheme of [29] is based on the idea that the TEQ filtering and the demodulating DFT can be interchanged. The equalizer is implemented *after* the DFT, hence it can be considered as "frequency domain" equalization. This allows for a decoupling of the equalizer design per tone i, with the advantage that the PTEQ with $L_w + 1$ taps per tone performs as well as and usually better (in terms of bit rate) than a single TEQ with $L_w + 1$ taps, with comparable complexity during data transmission. The idea behind the PTEQ scheme can be summarized compactly by noting that for a TEQ, the equalized i-th DFT output can be obtained in two ways:

$$U_i^k = \mathbf{f}_i(\mathbf{Y}^k\mathbf{w}) \quad = \quad (\mathbf{f}_i\mathbf{Y}^k)\mathbf{w}. \tag{54}$$

Here, $\mathbf{Y}^k$ is an $N \times (L_w + 1)$ Toeplitz matrix of received samples y_l^k of the current symbol k,

$$\mathbf{Y}^k \quad = \quad \begin{bmatrix} y_0^k & \cdots & y_{-L_w}^k \\ \vdots & \ddots & \vdots \\ y_{N-1}^k & \cdots & y_{N-L_w-1}^k \end{bmatrix}. \tag{55}$$

The left-hand side of (54) represents the classical convolution of the received signal y with the TEQ, $\mathbf{Y}^k\mathbf{w}$, followed by the DFT (Fig. 1). The right-hand side of (54) implies that the equalized i-th DFT output U_i^k can also be seen as a linear combination by $\mathbf{w}$ of $L_w + 1$ consecutive outputs of a sliding FFT on i-th tone, applied to the unequalized, received signal y_l^k.

A symbol estimate $\hat{X}_i^k$ (tone i, symbol k) is then obtained as

$$\hat{X}_i^k = (\mathbf{f}_i\mathbf{Y}^k)\underbrace{\mathbf{w}D_i}_{\mathbf{w}_i}, \tag{56}$$

where now a tone-dependent and complex set of coefficients $\mathbf{w}_i$ has been introduced by combining the TEQ $\mathbf{w}$ and the FEQ D_i. To avoid the need for $L_w + 1$ consecutive FFT operations

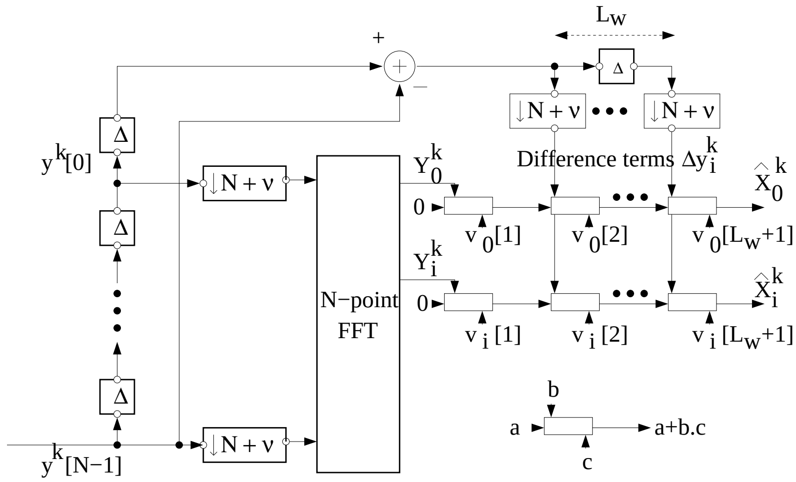

Fig. 4. PTEQ architecture: Channel Equalization Block at the Receiver

per symbol $\mathbf{f}_i\mathbf{Y}^k$ in (56), the Toeplitz structure of $\mathbf{Y}_k$ can be exploited:

$$\mathbf{f}_i\mathbf{Y}^k[:,l+1] \;=\; \alpha^{i-1}\mathbf{f}_i\mathbf{Y}^k[:,l] + \underbrace{(y^k_{-l} - y^k_{-l+N})}_{\Delta y^k_l}, \quad l = 1,\cdots,L_w. \tag{57}$$

Here, $\alpha = \exp(-j2\pi/N)$ and $\mathbf{Y}^k[:,l]$ denotes the l-th column of $\mathbf{Y}^k$. In other words, the DFT of a column of $\mathbf{Y}^k$ can be derived from the DFT of its previous column plus some correction term. An efficient implementation of (56) then only needs 1 FFT per symbol. The symbol estimate $\hat{X}^k_i$ is obtained by linearly combining the unequalized i-th DFT output Y^k_i with L_w real difference terms $\Delta y^k_l, l = 1,\cdots,L_w,$ as defined in (57):

$$\hat{X}^k_i \;=\; \underbrace{\left[\begin{array}{cccc} Y^k_i, & \Delta y^k_1, & \cdots, & \Delta y^k_{L_w}\end{array}\right]}_{\mathbf{z}^k_i}\mathbf{v}_i. \tag{58}$$

Here, $\mathbf{v}_i$ are the complex PTEQ coefficients, related to $\mathbf{w}_i$ in (56), by

$$\mathbf{v}_i[l] \;=\; \alpha^{i-1}\mathbf{v}_i[l+1] + \mathbf{w}_i[l], \qquad l = 1,\cdots,L_w \tag{59}$$

$$\mathbf{v}_i[L_w+1] \;=\; \mathbf{w}_i[L_w+1]. \tag{60}$$

Fig. 4 depicts the PTEQ scheme. An alternate derivation based on an infinite-impulse response (IIR) channel model in [67] led to a generalized PTEQ which exploits pilot and unused tones.

To determine a bit rate maximizing set of PTEQ coefficients $\mathbf{v}_i$ for each subchannel, it suffices to solve a linear MMSE cost function for each tone:

$$\min_{\mathbf{v}_i} J(\mathbf{v}_i) \;=\; \min_{\mathbf{v}_i} \mathcal{E}\left\{\left|\mathbf{Z}_i^k \mathbf{v}_i - X_i^k\right|^2\right\} \tag{61}$$

There are several initialization strategies for (61):

- the classical MMSE solution is given by

$$\mathbf{v}_i \;=\; \mathcal{E}\{(\mathbf{Z}_i^k)^H \mathbf{Z}_i^k\}^{-1} \mathcal{E}\{(\mathbf{Z}_i^k)^H (X_i^k)\} \tag{62}$$

- a generalized eigenvalue problem (3) could be solved for each tone i with [68]

$$\mathbf{A}_i \;=\; \mathcal{E}\{(\mathbf{Z}_i^k)^H \mathbf{Z}_i^k\} \tag{63}$$

$$\mathbf{B}_i \;=\; \mathcal{E}\{(\mathbf{Z}_i^k)^H (X_i^k)\}\mathcal{E}\{\mathbf{Z}_i^k (X_i^k)^*\}, \tag{64}$$

  which is equivalent to the MMSE solution;
- solving a least squares problem per tone as in [29], based on channel and noise estimates;
- an efficient blind or training-based adaptive algorithm [69], [70].

B. Time domain equalizer filter bank

An alternative scheme with an equalizer for each subchannel is the TEQ Filter Bank (TEQFB) [37]. The derivation writes the subchannel SNR as a single generalized Rayleigh quotient

$$\mathrm{SNR}_i = \frac{\mathbf{w}^T \tilde{\mathbf{B}}_i \mathbf{w}}{\mathbf{w}^T \tilde{\mathbf{A}}_i \mathbf{w}}, \tag{65}$$

where

$$\tilde{\mathbf{A}}_i \;=\; 2S_{x,i}\left(\mathbf{H}_{wall,1}^T \mathbf{V}_i \mathbf{V}_i^H \mathbf{H}_{wall,1} + \mathbf{H}_{wall,2}^T \mathbf{W}_i \mathbf{W}_i^H \mathbf{H}_{wall,2}\right) \tag{66}$$

$$+\mathbf{Q}_i^{\mathrm{noise}} \mathbf{R}_n \left[\mathbf{Q}_i^{\mathrm{noise}}\right]^H + \frac{\sigma_{\mathrm{DNF}}^2}{\mathbf{w}^T \mathbf{w}} \mathbf{I}_{L_w+1},$$

$$\tilde{\mathbf{B}}_i \;=\; S_{x,i} \mathbf{H}^T \mathbf{Q}_i^{\mathrm{circ}} \left[\mathbf{Q}_i^{\mathrm{circ}}\right]^H \mathbf{H}. \tag{67}$$

$\mathbf{H}_{wall,1}$ and $\mathbf{H}_{wall,2}$ are as in (20) and (22); $\mathbf{V}_i$ and $\mathbf{W}_i$ are upper and lower triangular Hankel matrices made from the ith row of the DFT matrix, $\mathbf{f}_i$; $\mathbf{Q}_i^{\mathrm{noise}}$ and $\mathbf{Q}_i^{\mathrm{circ}}$ are Hankel matrices made from $\mathbf{f}_i$; $\mathbf{R}_n$ is the noise (AWGN and crosstalk) covariance matrix; and σ_{DNF}^2 is the power of the noise due to the digital noise floor. See [37] for full definitions. The dependence of the number of bits per symbol on the TEQ is then established using (65):

$$\begin{aligned} b_{\text{DMT}}^{\text{int}}(\mathbf{w}) &= \sum_{i \in \mathcal{S}} \left\lfloor \log_2 \left(1 + \frac{\text{SNR}_i}{\Gamma_i}\right) \right\rfloor \\ &= \sum_{i \in \mathcal{S}} \left\lfloor \log_2 \frac{\mathbf{w}^T(\Gamma_i \tilde{\mathbf{A}}_i + \tilde{\mathbf{B}}_i)\mathbf{w}}{\mathbf{w}^T(\Gamma_i \tilde{\mathbf{A}}_i)\mathbf{w}} \right\rfloor \\ &= \sum_{i \in \mathcal{S}} \underbrace{\left\lfloor \log_2 \left(\frac{\mathbf{w}^T \mathbf{B}_i \mathbf{w}}{\mathbf{w}^T \mathbf{A}_i \mathbf{w}}\right) \right\rfloor}_{b_i^{\text{int}}(\mathbf{w})} \end{aligned} \tag{68}$$

Here, $\mathbf{B}_i = \Gamma_i \tilde{\mathbf{A}}_i + \tilde{\mathbf{B}}_i$; $\mathbf{A}_i = \Gamma_i \tilde{\mathbf{A}}_i$; $\lfloor . \rfloor$ is the flooring operation; Γ_i is the SNR gap, which is a function of the desired error probability, coding gain and system margin [8]; and $\mathcal{S}$ is the set of subchannels that carry data. The constraint $\mathbf{w}^T\mathbf{w} = 1$ is used in [37] to remove the dependence of the last term of $\tilde{\mathbf{A}}$ on $\mathbf{w}$, turning the argument of (68) into a generalized Rayleigh quotient.

The TEQFB then follows from (68) by maximizing each term b_i^{int} of the summation separately, as this maximizes the sum. For each subchannel i, a TEQ $\mathbf{w}_i$ is designed by solving

$$\mathbf{w}_i^{\text{opt}} = \arg \max_{\mathbf{w}_i : \|\mathbf{w}_i\|^2 = 1} \left(\frac{\mathbf{w}_i^T \mathbf{B}_i^r \mathbf{w}_i}{\mathbf{w}_i^T \mathbf{A}_i^r \mathbf{w}_i}\right), \tag{69}$$

where $(\cdot)^r$, denoting the real part, follows from the assumption that the TEQs $\mathbf{w}_i$ are real; and the flooring operation and the $\log_2$ function have been dropped as these do not change the solution $\mathbf{w}_i^{\text{opt}}$. The optimum bit allocation is then given by

$$\left[b^{\text{int}}\right]^{\text{opt}} = \sum_{i \in \mathcal{S}} \left\lfloor \log_2 \lambda_i^{\text{opt}} \right\rfloor, \tag{70}$$

where λ_i^{opt} is the largest generalized eigenvalue:

$$\lambda_i^{\text{opt}} = \frac{\left(\mathbf{w}_i^{\text{opt}}\right)^T \mathbf{B}_i^r \mathbf{w}_i^{\text{opt}}}{\left(\mathbf{w}_i^{\text{opt}}\right)^T \mathbf{A}_i^r \mathbf{w}_i^{\text{opt}}} = \frac{\left(\mathbf{w}_i^{\text{opt}}\right)^T \mathbf{B}_i \mathbf{w}_i^{\text{opt}}}{\left(\mathbf{w}_i^{\text{opt}}\right)^T \mathbf{A}_i \mathbf{w}_i^{\text{opt}}}. \tag{71}$$

Fig. 5 shows a block diagram of a DMT receiver with the TEQFB structure. Each TEQ $\mathbf{w}_i$ filters the received signal y^k. All $N \times 1$ TEQ output vectors $\mathbf{u}_i^k$ (with the cyclic prefix removed) are fed into a bank of Goertzel filters [71], where each Goertzel filter is tuned to the frequency of the desired subchannel and computes a single point DFT: $\mathbf{f}_i \mathbf{u}_i^k = \mathbf{f}_i(\mathbf{Y}^k \mathbf{w}_i)$ (with $\mathbf{Y}^k$ as defined in (55)). Finally, a 1-tap FEQ D_i is applied to each DFT output to give a symbol estimate $\hat{X}_i^k$.

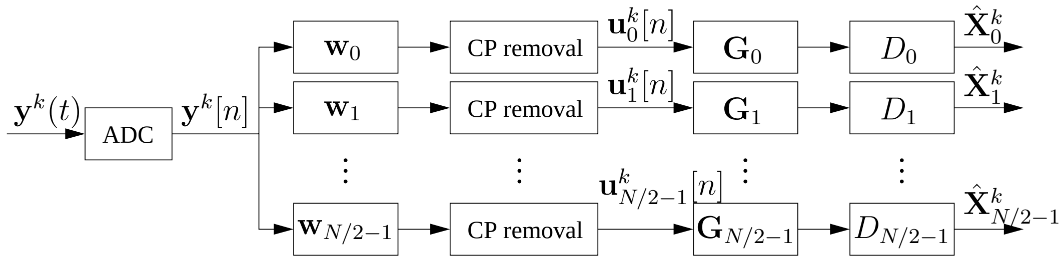

Fig. 5. TEQFB architecture: channel equalization block at the Receiver.

C. PTEQ or TEQFB?

The TEQFB in [37] is based on an approximate SNR model (65) whereas the PTEQ in [29] optimizes the true subchannel SNRs. Provided that the same *a priori* knowledge about channel and noise is used, an exact SNR model for the TEQFB is applied, and complex-valued TEQs are allowed, then the TEQFB and the PTEQ give the same performance, which is an upper bound for the single-TEQ-based receiver. Due to its large computational burden, the TEQFB in [37], [38] is not proposed as a practical approach, but as a bound to the performance that can be achieved with a single FIR TEQ. On the other hand, the PTEQ has roughly the same computational complexity as a TEQ-based receiver during data transmission, though the PTEQ and TEQFB both have high training complexity.

V. MORE THAN ONE RAYLEIGH QUOTIENT

This section discusses TEQ designs that attempt to optimize the bit rate by modelling it as a sum of logs of generalized Rayleigh quotients of the TEQ. This is equivalent to maximizing a product of generalized Rayleigh quotients, as in (1). The main distinctions between these designs are the approximations that are made and the progressively more rigorous models of the subchannel SNRs. Whereas the methods in the previous section optimized the bit rate separately on each tone, the methods in this section use a *single TEQ* to optimize the bit rate of the entire system, thereby leading to more complicated designs.

A. Maximum geometric signal-to-noise ratio (MGSNR) method

Al-Dhahir and Cioffi [16], [17], [43], [72], were the first to attempt bit rate maximization. Their approach was based on maximizing the geometric SNR (GSNR). Let B_i, W_i, and H_i

be the complex valued frequency domain representations of the TIR $\mathbf{b}$, the TEQ $\mathbf{w}$, and the transmission channel $\mathbf{h}$ in subchannel i, respectively. Then the SNR in subchannel i, assuming equal signal power distribution in all subchannels, is

$$\begin{aligned} \mathrm{SNR}_i &= \frac{S_x|H_i|^2}{S_{n,i}} = \frac{S_x|H_i|^2|W_i|^2}{S_{n,i}|W_i|^2} \\ &\cong \frac{S_x|B_i|^2}{S_{n,i}|W_i|^2} \end{aligned} \tag{72}$$

where $S_{n,i}$ and S_x are the noise and signal powers in subchannel i, respectively. The geometric SNR is then defined as

$$\begin{aligned} \mathrm{SNR}_{geom} &\triangleq \Gamma\left\{\left[\prod_{i\in\mathcal{S}}\left(1+\frac{\mathrm{SNR}_i}{\Gamma}\right)\right]^{\frac{1}{N_u}}-1\right\} \\ &\approx \prod_{i\in\mathcal{S}}(\mathrm{SNR}_i)^{\frac{1}{N_u}} \\ &= S_x\left[\prod_{i\in\mathcal{S}}\left(\frac{|B_i|^2}{S_{n,i}|W_i|^2}\right)\right]^{\frac{1}{N_u}} \end{aligned} \tag{73}$$

Here, N_u is the size of the set of used carriers, $\mathcal{S}$. Several simplifying assumptions are made. It is assumed that $SNR_i \gg \Gamma$ for all i, so that the unity terms in (73) can be ignored. However, this is not true in subchannels with low SNR. Also, the subchannel SNR definition does not include the effects of the ISI, ICI, and DFT leakage in the denominator, but instead only the power of the noise after the equalizer. The definition of the geometric SNR (72) also assumes that

$$\mathbf{f}_i\left(\mathbf{w}\star\mathbf{h}\right) = \mathbf{f}_i\mathbf{w}\ \mathbf{f}_i\mathbf{h} = W_iH_i \quad \text{and} \quad B_i = W_iH_i, \tag{74}$$

where $\star$ is time domain linear convolution and $\mathbf{f}_i$ is the i-th DFT row (assumed to be truncated to the length of $\mathbf{w}$ or $\mathbf{h}$). *Linear* convolution of $\mathbf{h}$ and $\mathbf{w}$ may not be equal to their product in the frequency domain and the difference appears as a noise source. These assumptions tend to design a TEQ that ignores the subchannels with lower SNR (which contain ISI and ICI that the TEQ is to remove) in favor of the subchannels with higher SNR, which does not maximize the data rate [34].

Under these assumptions, using (68) and (73), the DMT bit rate is approximately given by

$$b(\mathbf{w}) = N_u \log_2\left(1+\frac{\mathrm{SNR}_{geom}}{\Gamma}\right). \tag{75}$$

Maximizing (73) maximizes (75) as the logarithmic function is monotonically increasing. Maximizing (73) is *approximately* equivalent to maximizing the log of its numerator [43],

$$L(\mathbf{b}) = \frac{1}{N_u} \sum_{i \in \mathcal{S}} \ln |B_i|^2 = \frac{1}{N_u} \sum_{i \in \mathcal{S}} \ln \left(\mathbf{b}^T \mathbf{G}_i \mathbf{b} \right), \quad (76)$$

where $\mathbf{G}_i = \mathbf{g}_i \mathbf{g}_i^H$, and $\mathbf{g}_i^H$ consists of the first $\nu + 1$ elements of $\mathbf{f}_i$, the i-th row of the DFT matrix. The independence of the noise and the time domain equalizer $\mathbf{w}$ on $\mathbf{b}$ is assumed in (76), which is not correct as $\mathbf{w}$ is a function of $\mathbf{b}$ from (7).

The unit norm constraint is imposed on $\mathbf{b}$; however, according to [43], it then follows that $|B_i|^2 = 1$ for each i. This leads to a zero forcing solution for the time domain equalizer $\mathbf{w}$. Zero forcing is not necessary in DMT since it uses a guard band. To avoid the zero forcing solution a constraint is imposed that the MSE in (5) needs to be less than some value $\mathrm{MSE}_{\max}$. The optimal TIR $\mathbf{b}$ in terms of the maximum geometric SNR algorithm is then found by

$$\mathbf{b}_{\mathrm{GSNR}}^{\mathrm{opt}} = \arg \max_{\mathbf{b}} L(\mathbf{b}) = \arg \max_{\mathbf{b}} \prod_{i \in \mathcal{S}} \mathbf{b}^T \mathbf{G}_i \mathbf{b} \quad (77)$$
$$\text{s. t. } \mathbf{b}^T \mathbf{b} = 1, \text{ and } \mathbf{b}^T \mathbf{R}_\Delta \mathbf{b} < \mathrm{MSE}_{\max,}$$

where $\mathbf{R}_\Delta = \mathbf{A}$ from in (13). Note that (77) is equivalent to (1) with $\mathbf{B}_i = \mathbf{G}_i$ and $\mathbf{A}_i = \mathbf{I}_{\nu+1}$, but with an extra inequality constraint. Currently, this non-linear optimization problem can only be solved using numerical methods. Once the optimal TIR is found, the corresponding TEQ $\mathbf{w}$ follows from (7). Al-Dhahir and Cioffi [17] change the subchannel SNR model in (72) to include partially the effects of the ISI, but only when *evaluating* the TEQ designed using (72). An iterative GNSR maximization method was presented in [73].

B. Maximum bit rate (MBR) method

Arslan, Evans, and Kiaei [34], [74] proposed the Maximum Bit Rate (MBR) TEQ design, which follows the methods of partitioning the transmission channel impulse response of [32] and the subchannel SNR definition used in [17], [43], [72], to define a new model for the subchannel SNR. The subchannel SNR is modelled as in (37), leading to

$$\tilde{\mathbf{A}} = S_{n,i} \ \mathbf{f}_i^H[0 : L_w] \ \mathbf{f}_i[0 : L_w] + S_x \mathbf{H}^T \ \mathbf{D} \ \mathbf{f}_i^H \ \mathbf{f}_i \ \mathbf{D}\mathbf{H} \quad (78)$$

$$\tilde{\mathbf{B}} = S_x \mathbf{H}^T \mathrm{diag}(\mathbf{g}) \ \mathbf{f}_i^H[0 : L_w] \ \mathbf{f}_i[0 : L_w] \ \mathrm{diag}(\mathbf{g}) \ \mathbf{H}. \quad (79)$$

Then the bit rate can be modelled as a sum of logs of generalized Rayleigh quotients, proceeding as in (68) to obtain $\mathbf{A}_i = \Gamma_i \tilde{\mathbf{A}}_i$ and $\mathbf{B}_i = \Gamma_i \tilde{\mathbf{A}}_i + \tilde{\mathbf{B}}_i$. In (37), the numerator contains only the portion of the resulting transmission channel that contributes to the useful signal as opposed to (72) where the numerator contains the contribution of the entire channel. In (37), the denominator limits the contribution of the ISI noise to the shortened channel impulse response outside of the desired window, as opposed to the MGSNR model of (72), which ignores ISI-induced noise. The improvement over the MSSNR method [32] is that the subchannel SNR is defined in the frequency domain, thus enabling the design of a TEQ for a particular frequency band as opposed to MSSNR which cannot discriminate between subchannels. The subchannel SNR model in (37) includes ISI and additive Gaussian noise and is substituted into (68) to determine the bit rate. Optimization of the MBR model of the bit rate is a computationally intensive problem, and it will be discussed in Part II of this paper.

C. *Maximum data rate (MDR) TEQ*

Milosevic *et al.* [37] improved on the MBR method by forming a more rigorous model of the subchannel SNR. This model was presented in Section IV-B in the context of the TEQFB. The model includes near-end crosstalk, AWGN, analog-to-digital converter quantization noise, and the digital noise floor due to finite precision arithmetic. They also redefined the subchannel SNR model so that the desired signal is formed as the circular convolution of the data symbol with the channel impulse response at the input of the FFT (based on the minor approximation of a perfectly shortened channel), and the noise is the difference between the received and the desired signal. For a single TEQ, Milosevic *et al.* [37] simplify (68) by removing the flooring function, as in [34], [74]. The simplified function is the fractional number of bits per symbol,

$$b(\mathbf{w}, \mathcal{S}) \quad = \quad \sum_{i \in \mathcal{S}} \log_2 \left(\frac{\mathbf{w}^T \mathbf{B}_i \mathbf{w}}{\mathbf{w}^T \mathbf{A}_i \mathbf{w}} \right), \tag{80}$$

with $\mathbf{A}_i$ and $\mathbf{B}_i$ as in (66), (67), and (68). Empirical evidence suggests that the TEQs that maximize (68) and (80) may often be identical [37]; however, there is no guarantee that that is the case in general.

Equation (80) is again a sum of logarithms of ratios. Sum-of-ratios maximization is an active research topic in the fractional programming community for which no definitive solution exists yet (see e.g. [75], [76])[2]. Milosevic *et al.* [37] use modifications of Almogy and Levin's approach to the sum-of-ratios problem [77] to optimize (80); the details of this method will be discussed in Part II of this paper.

D. Bitrate maximizing TEQ (BM-TEQ)

Whereas the previous TEQ designs formulate a subchannel SNR model at the FFT output, Vanbleu *et al.* [44], [45] propose an improved/exact subchannel SNR. It is defined at the FEQ output by exploiting the dependence of the FEQs on the TEQ coefficients. The resulting SNR model is a nonlinear function of the TEQ coefficients which now accounts for the function of the FEQ, as well. The output of the FEQ is modelled as

$$D_i \underbrace{U_i^k}_{\mathbf{f}_i(\mathbf{Y}^k\mathbf{w})} = \alpha_i X_i^k + E_i^k \tag{81}$$

where D_i is the FEQ coefficient for tone i, U_i^k is the FEQ input, α_i is a bias, due to the equalizer, and E_i^k is the noise remaining on tone i. We assume unbiased MMSE FEQs

$$D_i = \frac{\mathcal{E}\{|X_i^k|^2\}}{\mathcal{E}\{(X_i^k)^* U_i^k\}}. \tag{82}$$

hence α_i is 1 and E_i^k contains all noise sources, including residual ISI/ICI and crosstalk. The dependence of the FEQs on the TEQ leads to the subchannel SNR model

$$SNR_i = \frac{\mathcal{E}\{|X_i^k|^2\}}{\mathcal{E}\{|D_i U_i^k - X_i^k|^2\}} = \frac{1}{\rho_i^{-2} - 1} \tag{83}$$

where

$$\rho_i^2 = \frac{|\mathcal{E}\{(X_i^k)^* U_i^k\}|^2}{\mathcal{E}\{|X_i^k|^2\}\mathcal{E}\{|U_i^k|^2\}}. \tag{84}$$

Substituting (83) into the bit rate equation (68) and exploiting (54), which was also used to derive the PTEQ in Section IV, we obtain the form of (80) in which the bit rate is a nonlinear function of the TEQ coefficients, with

$$\mathbf{A}_i = \Gamma_i \left(\mathcal{E}\left\{ \left| X_i^k \right|^2 \right\} \mathcal{E}\left\{ \left(\mathbf{Y}^k \right)^H \mathbf{f}_i^H \mathbf{f}_i \mathbf{Y}^k \right\} - \mathcal{E}\left\{ \left(\mathbf{Y}^k \right)^H \mathbf{f}_i^H X_i^k \right\} \mathcal{E}\left\{ \left(X_i^k \right)^H \mathbf{f}_i \mathbf{Y}^k \right\} \right) \tag{85}$$

$$\mathbf{B}_i = \Gamma_i \mathcal{E}\left\{ \left| X_i^k \right|^2 \right\} \mathcal{E}\left\{ \left(\mathbf{Y}^k \right)^H \mathbf{f}_i^H \mathbf{f}_i \mathbf{Y}^k \right\} + (1 - \Gamma_i)\, \mathcal{E}\left\{ \left(\mathbf{Y}^k \right)^H \mathbf{f}_i^H X_i^k \right\} \mathcal{E}\left\{ \left(X_i^k \right)^H \mathbf{f}_i \mathbf{Y}^k \right\} \tag{86}$$

where $\mathbf{Y}^k$ is as in (55). Maximization of this function will be discussed in Part II of this paper.

[2]However, (80) is a sum of logarithms of ratios, or a log of a product of ratios as in (1), thus maximizing it is an even more involved problem that of maximizing a sum of ratios.

VI. Exceptions to the common formulation

This section addresses two single TEQ design methods that do not fit in the common framework of Section II. The first method (by Henkel and Kessler) attempts to iteratively design a TEQ that maximizes the bit rate, hence its performance should be compared with the algorithms of Section V. The second method describes a channel shortening criterion, based on a *sum* rather than a *product* of generalized Rayleigh quotients in (1). It is meant to be solved blindly.

A. Henkel and Kessler's bit rate maximizing iterative algorithm

Henkel and Kessler describe in [78] one of the first attempts to improve upon the TEQ design of Al-Dhahir and Cioffi in [43]. They present a subchannel SNR model that includes the leakage effect of the DFT on the noise as well as ISI. The leakage effect comes from the implicit rectangular time-domain window (due to the DFT) or, alternatively, from a (twice as wide) triangular window applied to the autocorrelation function of noise and ISI. This autocorrelation function is obtained as the IDFT of the noise and ISI power spectral density, oversampled in the frequency domain by a factor of 2. ICI is neglected and the SNR updating algorithm assumes that all carriers are active. Based on this improved subchannel SNR model, any multidimensional optimization algorithm can be used to optimize the bitrate. Contrary to most other TEQ designs, the frequency response of the TEQ, rather than the time-domain version $\mathbf{w}$, is optimized, as the SNR model is derived in terms of the TEQ discrete frequency response. The subchannel SNR updating algorithm renders the method outside of our general framework (1).

B. Channel shortening by sum-squared auto-correlation minimization (SAM)

A recently proposed algorithm [79] meant for blind, adaptive channel shortening minimizes the sum-squared auto-correlation in terms of the effective channel impulse response $\mathbf{c} = \mathbf{h} \star \mathbf{w}$ outside a window of desired length $\nu + 1$:

$$\min_{\mathbf{w}} J(\mathbf{w}) \;\; = \;\; \sum_{l=\nu+1}^{L_c} |R_{\mathbf{c}}[l]|^2 \quad \text{subject to } \; \|\mathbf{w}\|^2 = 1 \tag{87}$$

where $R_{\mathbf{c}}[l] = \sum_{k=0}^{L_c} \mathbf{c}[k]\mathbf{c}[k-l]$ is the autocorrelation sequence of the effective channel $\mathbf{c} = \mathbf{h}\star\mathbf{w}$. The constraint $\|\mathbf{w}\|^2 = 1$ prevents the all-zero solution (other constraints are also possible). Note that the summation in (87) extends over terms outside the desired window of length $\nu + 1$.

The cost function is not meant to provide optimal channel shortening in terms of, e.g., maximum shortening SNR or maximum bit rate. The rationale behind it is the fact that this cost function gives rise to an elegant blind, adaptive channel shortening algorithm under the assumption that the transmit sequence is white and wide-sense stationary. Under this assumption, the cost function can be well approximated as a function of the equalized TEQ output u_n,

$$J(\mathbf{w}) \approx \sum_{l=\nu+1}^{L_c} |\mathcal{E}\{u_n u_{n-l}\}|^2, \tag{88}$$

which can be solved using a *blind* stochastic gradient descent algorithm. As it is fourth-order in $\mathbf{w}$ (and hence multimodal), proper initialization is required. Interestingly, if the summation is replaced by a product in (87), the algorithm does fit into the formulation of (1).

Miyajima and Ding [80] have proposed a blind, non-adaptive autocorrelation-shortening TEQ based on oversampling (spatial or temporal), which fits into the Rayleigh quotient model of (2). We do not consider oversampling in this paper, but details can be found in Section 3.4 of [80].

VII. Conclusion

This paper presented an overview of existing design methods to tackle the equalization problem in multicarrier systems where the time span of the channel impulse response is longer than the cyclic prefix duration. The majority of these techniques can be cast into a common formulation based on a maximization of a product of generalized Rayleigh quotients. In addition, we provided a unified notational framework and an extensive literature survey. One goal of this unification and common formulation was to clarify the subtle differences between these methods.

Based on the common formulation, several categories were distinguished, each leading to a different design strategy. A single generalized Rayleigh quotient led to suboptimal (in terms of bit rate maximization) single TEQ designs. On the other hand, optimizing a single generalized Rayleigh quotient for each tone separately resulted in a bank of TEQ filters, which is a generalization of the single TEQ case. The more difficult case with a product of multiple generalized Rayleigh quotients is required for optimal single TEQ design. Of course, an entire class of intermediate designs could be devised based on multiple TEQ designs for subgroups of tones.

Each of these classes has its advantages and drawbacks. Therefore, this paper is accompanied with a second part that focuses on a performance comparison together with investigation of complexity and design issues when effectively implementing a multicarrier equalizer in hardware.

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