Low Complexity MIMO Blind Adaptive Channel Shortening

Richard K. Martin, John M. Walsh, and C. Richard Johnson, Jr.

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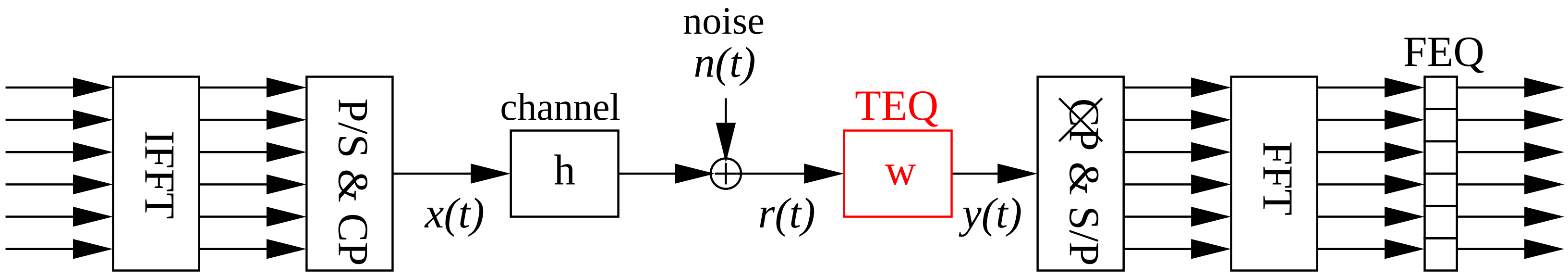
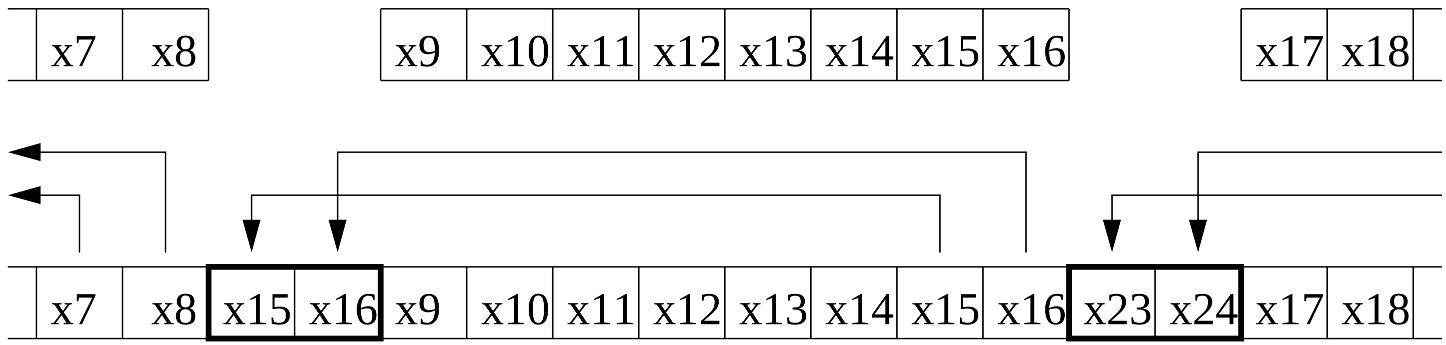
Multicarrier system model



- Analog viewpoint: signal is divided into bins. Bin i is modulated by a carrier at frequency $i \cdot f_c$, where $0 \le i \le N-1$.
- Digital viewpoint: efficient implementation by IFFT.
- Goal: change frequency selective channel into N parallel flat fading channels.
- A bank of complex scalars, a Frequency-domain Equalizer, (FEQ) inverts the flat fades.

What is the cyclic prefix (CP)?



- Before each block is transmitted, the CP is prepended.
- $\mathbf{H}$ is the channel convolution matrix ($\mathbf{y} = \mathbf{H}\mathbf{x}$)
- If we add the CP, $\mathbf{H} \rightarrow \widehat{\mathbf{H}}$, a circulant matrix.
- Fact: DFT matrices diagonalize any circulant matrix.

$$\mathcal{F} \; \widehat{\mathbf{H}} \; \mathcal{F}^H = \Lambda = \text{diagonal}$$

- All of this is only true if the CP+1 is as long as the channel.
- CP length is fixed. Sometimes the channel is longer.

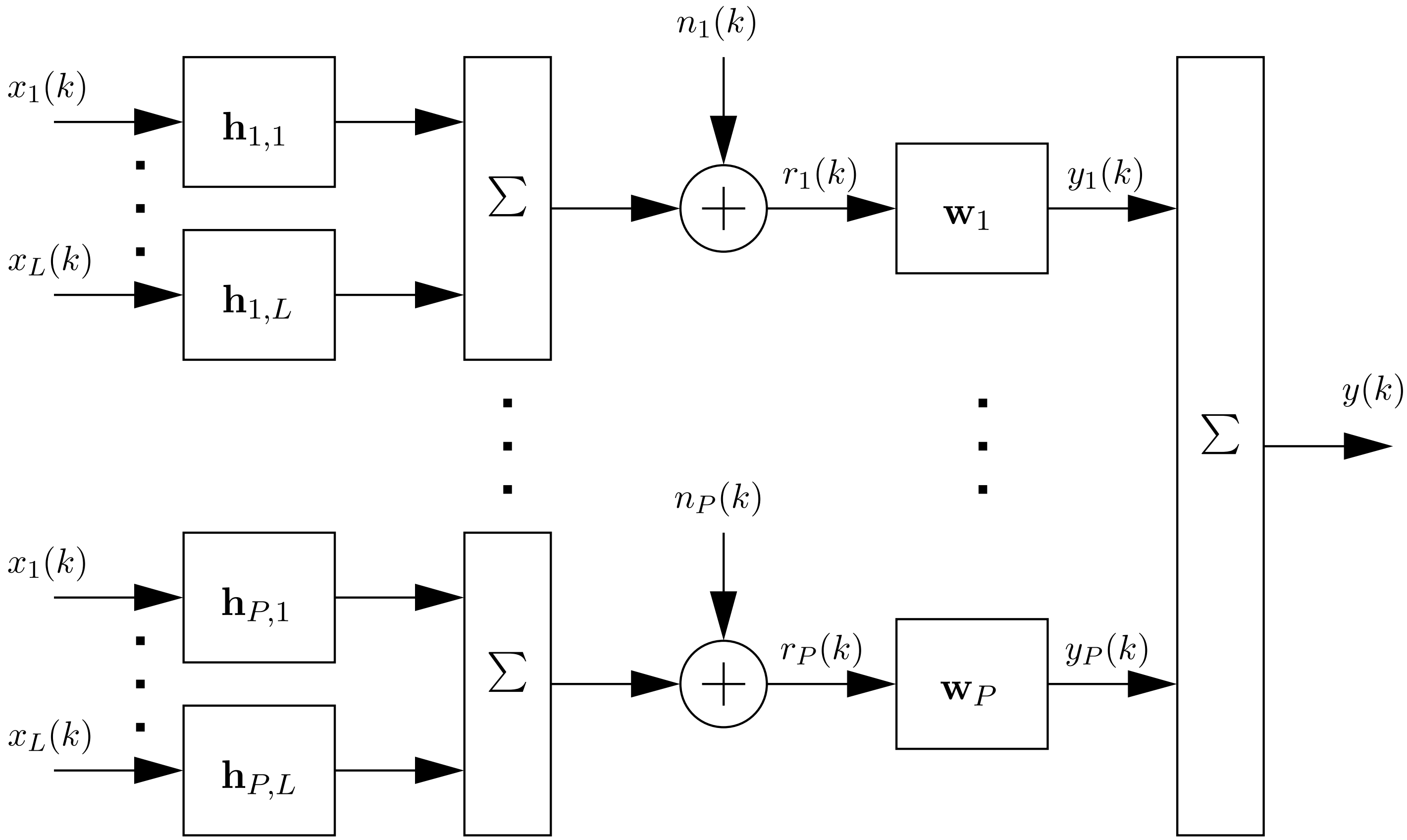
Parameter selection guidelines

1. The block size must be short enough to minimize the channel's time variations within each symbol. (Keep N small.)
2. The guard interval must be long enough to exceed the delay spread of most of the channels that will be encountered. (Keep ν large.)
3. The throughput loss due to the use of the guard interval must be kept as small as possible. (Keep ν much smaller than N.)

These guidelines cannot always be satisfied.

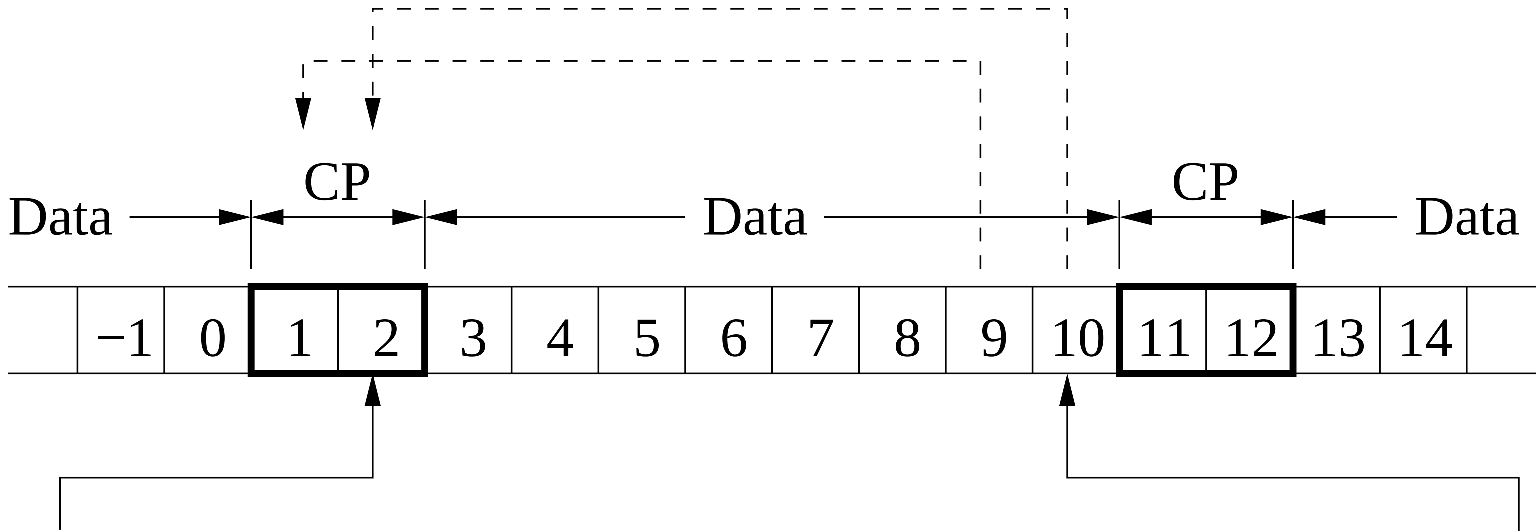
Solution: ignore guideline #2, and use a channel shortening filter.

MIMO channel shortening model



- L users or L transmit antennas
- P receive antennas or oversampling by P

MERRY: some intuition



$$
\begin{aligned}
y(2) &= c_0x(2) + c_1x(1) + c_2x(0) + c_3x(-1) + c_4x(-2) \\
&= c_0x(10) + c_1x(9) + [c_2x(0) + c_3x(-1) + c_4x(-2)] \\
y(10) &= c_0x(10) + c_1x(9) + [c_2x(8) + c_3x(7) + c_4x(6)]
\end{aligned}
$$

- The transmitted data in the CP and the end of the symbol are equal.
- If the channel is short enough, then the last sample in the received CP equals the last sample in the received symbol.

The MERRY algorithm

- Idea: try to force a particular sample in the CP to equal the corresponding sample in the symbol.
- Algorithm: perform a (stochastic) gradient descent of

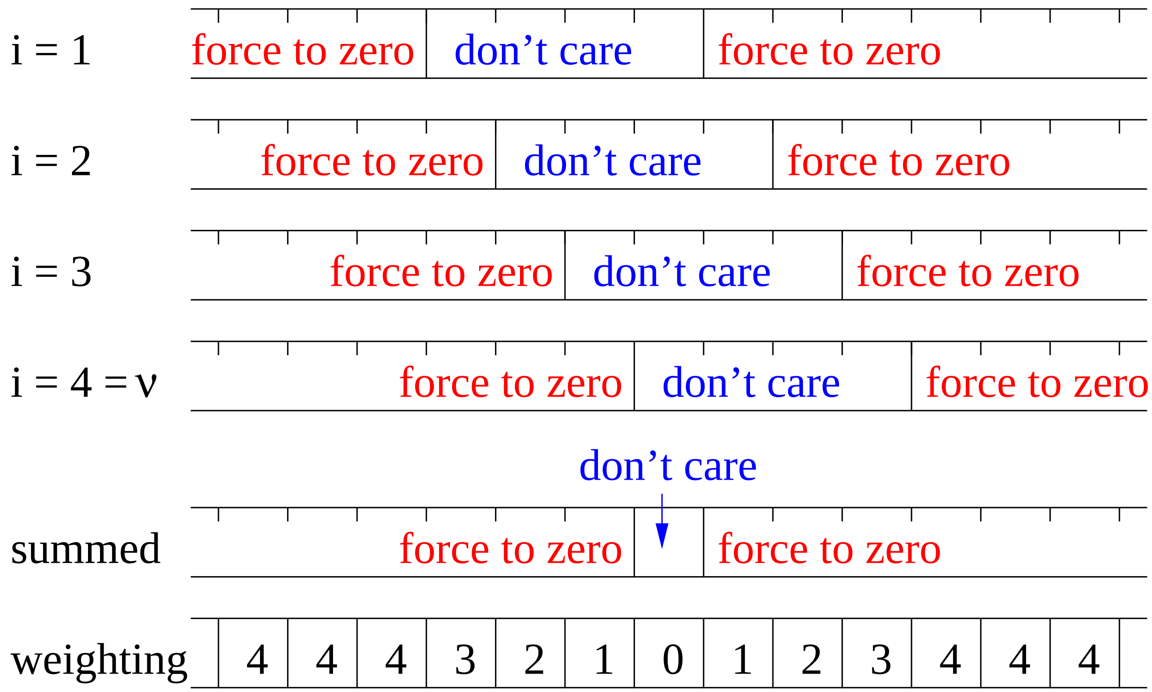
$$J_{MERRY} = \mathrm{E}\left[|y(\nu + \Delta) - y(\nu + N + \Delta)|^2 \right], \tag{1}$$

for some Δ in the set $\{0, \ldots, M-1\}$, where M is the symbol length.

- A constraint is needed to prevent $\mathbf{w} = \mathbf{0}$.
- Multicarrier Equalization by Restoration of RedundancY (MERRY):

$$\boxed{\begin{aligned} \tilde{\mathbf{r}}(k) &= \mathbf{r}(Mk + \nu + \Delta) - \mathbf{r}(Mk + \nu + N + \Delta) \\ e(k) &= \mathbf{w}^T(k)\ \tilde{\mathbf{r}}(k) \\ \hat{\mathbf{w}}(k+1) &= \mathbf{w}(k) - \mu\ e(k)\ \tilde{\mathbf{r}}^*(k) \\ \mathbf{w}(k+1) &= \frac{\hat{\mathbf{w}}(k+1)}{\|\hat{\mathbf{w}}(k+1)\|_2} \end{aligned}} \tag{2}$$

The FRODO cost function



- Why not form more than one "cyclic difference" in MERRY?
- Result: variable window size, formed by an intersection of up to ν different MERRY windows
- FRODO: Forced Redundancy with Optional Data Omission

Division-free update rule: preliminaries

- Instead of minimizing the "wall" with respect to the energy in the channel shortener, we can minimize with respect to the energy in the "window" of the effective channel.
- Recall that $\mathrm{E}\left[\left|y(\nu+\Delta)-y(\nu+N+\Delta)\right|^2\right]$ equals the "wall" power.
- It turns out that $\mathrm{E}\left[y^*(\nu+\Delta)y(\nu+N+\Delta)\right]$ is the "window" power.
- These two quantities are quadratic in the channel shortener, i.e. they equal $\mathbf{w}^T\mathbf{A}\mathbf{w}$ and $\mathbf{w}^T\mathbf{B}\mathbf{w}$.
- We want to solve

$$\mathbf{w}_{opt} = \arg\max_{\mathbf{w}} \frac{\text{window power}}{\text{wall power}} = \arg\max_{\mathbf{w}} \frac{\mathbf{w}^T\mathbf{B}\mathbf{w}}{\mathbf{w}^T\mathbf{A}\mathbf{w}} \tag{3}$$

FRODO: Division-free update rule

- Chatterjee, *et al.* proposed an iterative algorithm for computing the maximum generalized eigenvalue and its eigenvector:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \left(\mathbf{B}\mathbf{w} - \mathbf{A}\mathbf{w}\left(\mathbf{w}^H \mathbf{B}\mathbf{w}\right)\right) \tag{4}$$

- We can remove the expectations from $\mathbf{A}$ and $\mathbf{B}$ to get stochastic approximations. Combining this with Chatterjee's general algorithmic form, we get an adaptive generalized eigen solver.
- The new update rule is:

$$\boxed{\begin{array}{l} \text{Given } \Delta \text{ and } i_o, \text{ for symbol } k = 0, 1, 2, \ldots, \\ \qquad \tilde{\mathbf{r}}(k) = \mathbf{r}(Mk + \nu + \Delta) - \mathbf{r}(Mk + \nu + N + \Delta) \\ \qquad e(k) = \mathbf{w}^T(k)\ \tilde{\mathbf{r}}(k) \\ \qquad y_{i_o}(k) = y(Mk + i_o + \Delta) \\ \mathbf{w}(k+1) = \mathbf{w}(k) + \mu\ y_{i_o}(k)\left(\mathbf{r}^*(Mk + i_o + \Delta) - y_{i_o}^*(k) e(k) \tilde{\mathbf{r}}^*(k)\right) \end{array}}$$

Initialization

- We have

$$\mathbf{A} = \mathrm{E}\left[\left(\mathbf{r}_{\nu+\Delta}(k) - \mathbf{r}_{\nu+N+\Delta}(k)\right)\left(\mathbf{r}_{\nu+\Delta}(k) - \mathbf{r}_{\nu+N+\Delta}(k)\right)^T\right]$$
$$\mathbf{B} = \mathrm{E}\left[\mathbf{r}_{\nu+\Delta}(k)\ \mathbf{r}^H_{\nu+N+\Delta}(k)\right]$$

where k is the block number.

- We can initialize near the correct location by computing

$$\widehat{\mathbf{A}} = \frac{1}{K}\sum_{k=1}^{K}\left(\mathbf{r}_{\nu+\Delta}(k) - \mathbf{r}_{\nu+N+\Delta}(k)\right)\left(\mathbf{r}_{\nu+\Delta}(k) - \mathbf{r}_{\nu+N+\Delta}(k)\right)^T$$
$$\widehat{\mathbf{B}} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{r}_{\nu+\Delta}(k)\ \mathbf{r}^H_{\nu+N+\Delta}(k)$$

and then computing the extreme eigenpair of matrices $\left(\widehat{\mathbf{A}}, \widehat{\mathbf{B}}\right)$.

- For a good estimate, $K \approx L_w$ or greater.

Synchronization

- Hitherto, all equations presume a good choice of delay Δ.
- Proposed heuristic choice: if Δ_{peak} optimizes the energy in a window of the channel, then choose

$$\Delta = \Delta_{peak} + \left\lfloor \frac{L_w}{2} \right\rfloor \tag{5}$$

- To get Δ_{peak}, first compute

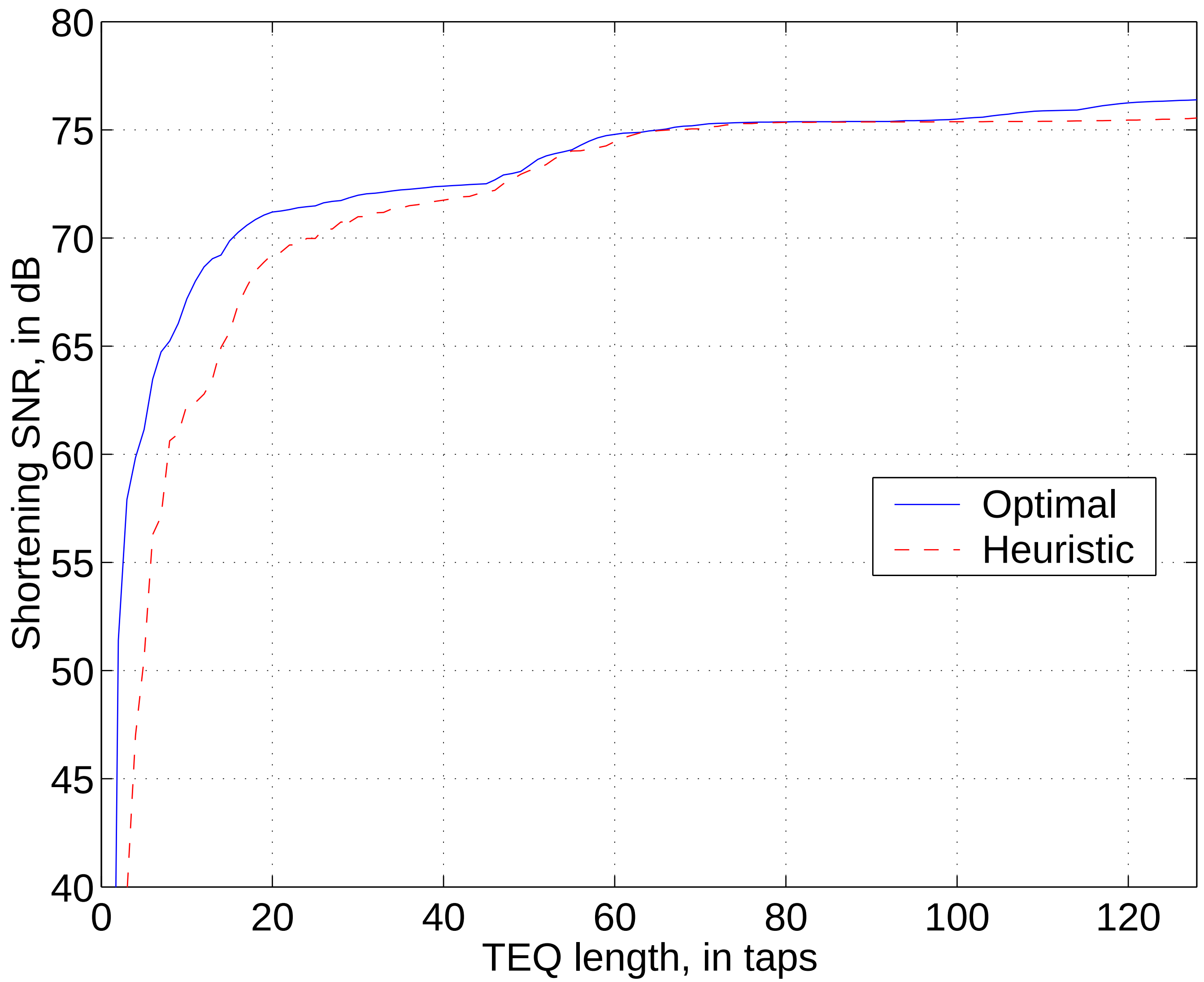
$$J_h(\Delta) = \sum_{k=1}^{K} |r(Mk + \nu + \Delta) - r(Mk + \nu + N + \Delta)|^2$$

  Like the MERRY cost, this gives the "wall" energy of the channel.

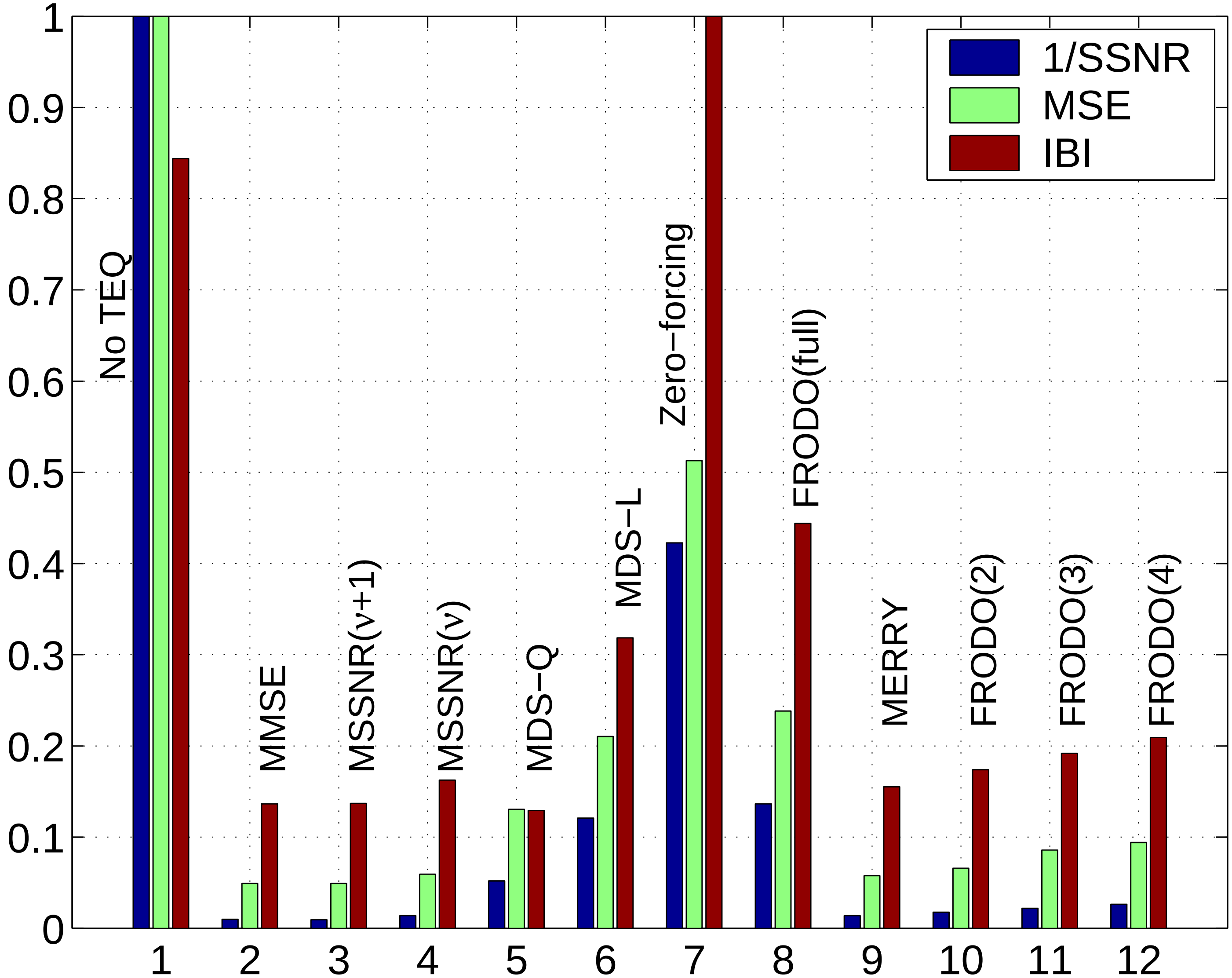
- The smaller the wall energy, the larger the window energy, so

$$\widehat{\Delta}_{peak} = \arg \min_{0 \le \Delta \le M-1} J_h(\Delta) \tag{6}$$

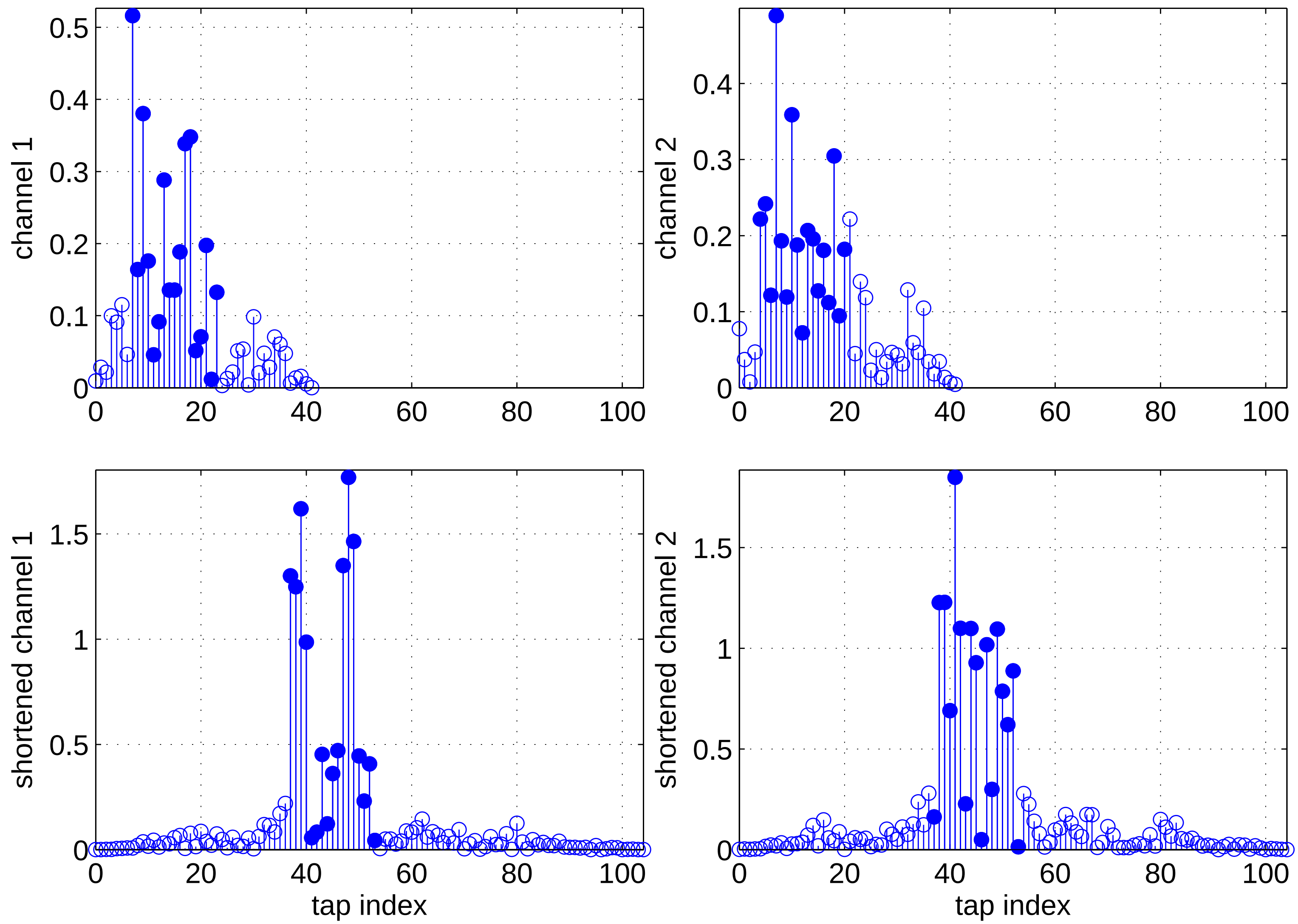
Performance of a heuristic delay choice



Design comparison



Performance of MIMO Merry



Conclusions

- We have extended MERRY in various ways, and dubbed the new algorithm "FRODO."
- FRODO avoids the computationally expensive square root and division of other channel shorteners, including MERRY.
- FRODO allows MIMO channel shortening and multiple comparisons between the CP and data.
- The use of more comparisons increases convergence speed but degrades asymptotic performance.
- We proposed blind initialization and synchronization techniques for a channel shortener.