Blind Carrier Frequency Offset Synchronization for OFDM Systems based on Higher Order Statistics

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Abstract — **We develop a new blind carrier frequency offset correction scheme for OFDM systems based on fourth order statistics. The proposed algorithm does not require the transmission of known signals and performs perfect frequency offset synchronization under a certain condition on the channel which is satisfied for virtually all OFDM channels in practical situations. The global convergence of this algorithm to a unimodal solution can be shown independently from the channel. Simulation results and the connection to other algorithms, such as Maximum Likelihood(ML) approximation and Constant Modulus Algorithm (CMA), are presented.**

I. Introduction

In OFDM systems, the importance of synchronization goes beyond the conventional concept in single carrier systems as an optimal conversion from the IF analog signal to the baseband digital signal. Proper synchronization in multicarrier systems is required to maintain orthogonality of subchannels before equalization and detection. Destruction of orthogonality of subcarriers results in inter channel interference (ICI) and inter symbol interference (ISI), and consequently degrades overall system performance. Therefore, preserving the orthogonality of the subchannels is considered a central task of signal processing for multicarrier systems just as equalization is for single carrier systems [1].

Carrier frequency offset is one of the critical synchronization parameters in preserving orthogonality [2] and abundant techniques have been developed for this task. Most existing carrier frequency offset recovery schemes rely on training sequences [3], [4], [5], [6], [7]. But, obviously, these approaches lower the achievable information rate. To avoid this loss, several methods that do not require pilot symbols or training sequences have been proposed. These blind approaches either exploit the cyclic prefix structure [8], [9], [10] or feedback from the FFT output. These feedback schemes can be further divided into an ML approximation approach [11], [12] or estimation-based approach using the second order statistics [13], [14]. However, the methods using the cyclic prefix structure exhibit serious performance degradation in the presence of multipath channels. One of the second order statistics approaches, the method exploiting existence of null subcarriers in most OFDM standards [14], is vulnerable to the interference in null subcarriers. The performance of the ESPRIT based algorithm [13], another second order statistics approach, heavily depends on the channel zero locations. On the other hand, the ML approximation approaches can overcome such drawbacks. However, ML approaches are derived under the assumption of an ideal AWGN channel and hence theoretical assertion for reliable performance in the presence of multipath channel and for global convergence is required. Furthermore, the first order approximation of ML function considered in [11] produces an almost flat cost function for discrete time implementation of OFDM, which results in unreliable performance.

In this paper, we propose a simple algorithm based on the fourth order statistics of the FFT output. Exploitation of higher order statistics has proven to be beneficial for blind equalization in single carrier system. [16] Our approach is an extension of blind equalization techniques for single carrier systems to OFDM system in order to minimize ICI caused by inaccurate frequency offset. In Section II, the system model for a discrete time OFDM is introduced and ICI due to frequency offset is formulated. Section III proposes a frequency estimation algorithm based on the fourth power accumulation and investigates its global convergence. In Section IV several connections to other approaches, such as ML approximations, are mentioned. Section V presents simulation results and concludes.

II. System Model

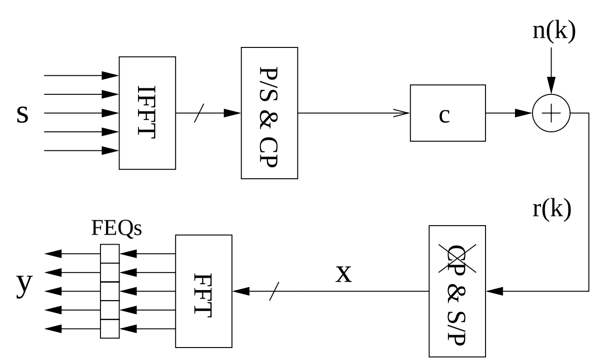

Figure 1: Discrete-time Multicarrier system model

Consider a discrete time baseband OFDM system as illustrated in Figure 1, where $\mathbf{s}_l = [s_{0,l} \cdots s_{N-1,l}]^t$ denotes l-th block of identically independently distributed (i.i.d.) source data drawn from a finite alphabet set. $\mathbf{s}_l$ is transformed via an inverse discrete Fourier Transformation (IDFT) and a cyclic-prefix of length L is inserted. Then, it is transmitted through a multipath channel with impulse response $c(k)$ and additive white Gaussian noise $w(k)$. $r(k)$ denotes the received signal

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and $\mathbf{x}_l = [x_{0,l} \cdots x_{N-1,l}]^t$ denotes the block of data after removal of the cyclic-prefix. $\mathbf{x}_l$ is now processed by a discrete Fourier Transformation (DFT) and finally the output signal $\mathbf{y}_l = [y_{0,l} \cdots y_{N-1,l}]$ is produced. Under the assumption that channel delay spread does not exceed the cyclic prefix interval (i.e. channel length $N_c \leq \mathrm{L}+1$) and symbol synchronization is properly done, the channel convolution matrix C in the following relation between source data and received data after cyclic prefix removal,

$$\mathbf{x}_l = CD^H\mathbf{s}_l + \mathbf{w}_l, \qquad (1)$$

becomes a circulant matrix, i.e.

$$C = \begin{pmatrix} c_0 & c_1 & c_2 & \cdots & c_L & 0 & \cdots & 0 \\ 0 & c_0 & c_1 & c_2 & \cdots & c_L & 0 & \cdots \\ & & \ddots & & \ddots & & \ddots & \\ c_2 & \cdots & c_L & 0 & \cdots & 0 & c_0 & c_1 \\ c_1 & c_2 & \cdots & c_L & 0 & \cdots & 0 & c_0 \end{pmatrix} \qquad (2)$$

where D denotes the DFT matrix, $\mathbf{w}_l$ denotes a vector of i.i.d. white complex circular Gaussian noises, which are uncorrelated to the source signals.

In the absence of carrier frequency offset, the input-output relation for the source $\mathbf{s}_l$ and the output $\mathbf{y}_l$ can be written as

$$\mathbf{y}_l = DCD^H\mathbf{s}_l + D\mathbf{w}_l$$

As a circulant matrix, C can be uniquely factorized as $C = D^H\Lambda D$, where Λ is a diagonal matrix, which represents the DFT of the channel $c(k)$ [15]. Therefore, DFT operation diagonalizes the channel matrix and as a result we have orthogonal subchannels

$$\mathbf{y}_l = \Lambda\mathbf{s}_l + D\mathbf{w}_l. \qquad (3)$$

However, in the presence of frequency offset ν normalized to the symbol rate $1/T$, the received signal $r(k)$ is multiplied by $e^{j2\pi\nu k/N}$ and

$$\mathbf{x}_l = F_\nu CD^H\mathbf{s}_l + F_\nu\mathbf{w}_l \qquad (4)$$

where F_ν is a diagonal matrix defined as $F_\nu[k,k] = e^{j2\pi\nu k/N}$ $(-1/2 < \nu < 1/2)$. Thus, the input-output relation can be written as

$$\begin{aligned} \mathbf{y}_l &= DF_\nu CD^H\mathbf{s}_l + DF_\nu\mathbf{w}_l \\ &= DF_\nu D^H\Lambda\mathbf{s}_l + DF_\nu\mathbf{w}_l \end{aligned}$$

Due to the non-trivial F_ν, $DF_\nu D^H$ cannot be reduced to a diagonal matrix, and thus ICI is introduced:

$$\begin{aligned} y_{n,l}(\nu) &= \Lambda_n s_{n,l} I_0(\nu) e^{\pi j \frac{N-1}{N}\nu} \\ &\quad + \sum_{m \neq n} \Lambda_m s_{m,l} I_{m-n}(\nu) e^{\pi j \frac{N-1}{N}(m-n+\nu)} + w_n \end{aligned}$$

where w_n denotes the n-th component of $DF_\nu\mathbf{w}_l$ and $I_k(x)$ is a real valued function defined as

$$I_k(\nu) := \frac{1}{N}\left|\sum_{l=0}^{N-1} e^{2\pi j \frac{l}{N}(k+\nu)}\right| = \frac{\sin(\pi(k+\nu))}{N\sin(\pi(k+\nu)/N)} \qquad (5)$$

Notice that for a circular complex Gaussian $\mathbf{w}_l$, w_n is white $(E\{|w_n|^2\} = \sigma_w^2)$ and uncorrelated with other subchannel noises $(E\{w_n^* w_m\} = 0$ for $m \neq n)$.

The function I_k represents the effect of ν on ICI, since $I_k = \delta(k)$ if and only if $\nu = 0$. It is useful to study some properties of I_k in order to develop frequency offset correction algorithm in next section.

Property 1 We observe the following two properies of $I_k(\nu)$

$$i) \quad \sum_{k=0}^{N-1} I_k^2(\nu) = 1 \qquad (6)$$

$$ii) \quad \sum_{k=0}^{N-1} I_k^4(\nu) = 1 - \frac{2}{3}\left(1 - \left(\frac{1}{4}\right)^{\log_2 N}\right)\sin^2(\pi\nu), \qquad (7)$$

where in $ii)$, without loss of much generality, we assume N is a power of 2.

Proof: See Appendix A.

III. Frequency Estimation Algorithm

Consider the carrier frequency offset synchronization scheme illustrated in Figure 2. Output of FFT is fed back to the synchronization device to estimate carrier frequency offset ν and correct frequency offset symbol by symbol. In the sequel we will develop the algorithm in detail.

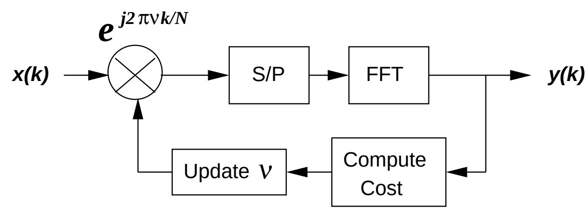

Figure 2: Block diagram of the proposed Carrier Frequency Offset Synchronization.

A. Cost Function

Property 1 says that $\sum_k I_k^4(\nu)$ is a unimodal function of ν, while $\sum_k I_k^2(\nu)$ is constant. Hoping we can exploit this property, we consider the following cost function based on the fourth order statistics of the output in order to minimize ICI terms in (5) and consequently correct frequency offset

$$J = \sum_{n=0}^{N-1} E\left\{|y_n(\nu)|^4\right\}. \qquad (8)$$

Expansion of the expectation will reveal the dynamics of this cost function. We need to introduce some notation for further investigation.

- *Kurtosis for a random sequence:* For a random sequence s, Kurtosis, κ_s is defined as

$$\kappa_s = \frac{E\left\{|s|^4\right\}}{E\left\{|s|^2\right\}^2} \qquad (9)$$

  Notice that for a Gaussian sequence $\{w\}$,

$$\kappa_w = \begin{cases} 3 & \text{when } \{w\} \text{ is real} \\ 2 & \text{when } \{w\} \text{ is complex} \end{cases} \qquad (10)$$

- *Kurtosis for a vector:* One can extend the definition of kurtosis for a vector x, by

$$\kappa_x = \frac{\|x\|_4^4}{\|x\|^4} \qquad (11)$$

where $\|\cdot\|_4$ is ℓ_4-norm defined as

$$\|x\|_4 = \sqrt[4]{\sum_i |x_i|^4} \quad (12)$$

Notice that for a length N vector x

$$\frac{1}{N} \leq \kappa_x \leq 1 \quad (13)$$

and κ_x has maximum 1 when all $N-1$ entries are zero and minimum $1/N$ when all N entries have the same absolute value.

With these notations, the proposed cost function can be written as a function of ν in the following way:

Theorem 1 Consider the diagonal channel frequency response matrix Λ as a vector and let ν denote the parameter error of the carrier frequency offset estimator. Then we have,

$$i) \sum_{n=0}^{N-1} E\left\{|y_n(\nu)|^2\right\} = \|\Lambda\|^2 + \sigma_w^2 \quad (14)$$

$$ii) \sum_{n=0}^{N-1} E\left\{|y_n(\nu)|^4\right\} = \|\Lambda\|^4 \kappa_s \kappa_\Lambda \quad (15)$$

$$- \frac{2\|\Lambda\|^4}{3}((\kappa_w + \kappa_s)\kappa_\Lambda - \kappa_w)(1 - (\frac{1}{4})^{log_2 N})\sin^2(\pi\nu) \quad (16)$$

$$+ 2\kappa_w \|\Lambda\|^2 \sigma_w^2 + \kappa_w \sigma_w^2$$

Proof: : See Appendix

Theorem 1 implies that the proposed cost function (8) has a unimodal minimum at $\nu = 0$ when

$$\kappa_\Lambda < \frac{\kappa_w}{\kappa_w + \kappa_s} \quad (17)$$

(for example, Figure 3 illustrates the cost function for an ideal channel) and a unimodal maximum at $\nu = 0$ otherwise. Since for most communication sources $1 \leq \kappa_s < \kappa_w$, it is enough to check if $\kappa_\Lambda < 1/2$ holds for most OFDM channels in order to justify minimization of the cost function (8). For now, we develop a heuristic argument for this. The kurtosis of a vector tends to indicate the "peakyness" of the vector (recall that the kurtosis of a vector has minimum at a vector with all equal absolute value components and maximum at a vector with only one nonzero component). Due to the assumption that channel length N_c is less than the cyclic prefix length L (which is less than the DFT size), Λ is given by interpolation of a length N_c vector and consequently the peakyness of Λ is limited by the percentage of the cyclic prefix length. Since $\Lambda = D\,[c_0\, c_1 \cdots c_{N_c-1}\, 0 \cdots 0]^t$, we assume that the flat time domain channel response $c = [1 \cdots 1]$ produces most "peaky" Λ in the frequency domain. Figure 4 plots κ_Λ versus the cyclic prefix length as a percentage of the symbol length for $c = [1 \cdots 1]$. κ_Λ does not exceed 0.2 for the cyclic prefix length less then 25% of the symbol length and achieves 0.5 for the cyclic prefix length larger than 60%. Usually the cyclic prefix length does not exceed 25% of the symbol length, and consequently the minimization of the cost function (8) is a successful strategy for correction of the carrier frequency offset ν.

B. Estimation Algorithm

We use the stochastic descent algorithm [17] of the cost function (8) to estimate the carrier frequency offset ν. The carrier offset correction parameter is updated each time by minimizing the instantaneous cost function, $\sum_n |y_{n,l}(\nu_k)|^4$, i.e.

$$\nu_{k+1} = \nu_k - \frac{\mu}{4}\frac{\partial}{\partial\nu}\sum_n |y_{n,l}(\nu_k)|^4 \quad (18)$$

$$= \nu_k - \mu \sum_n |y_{n,l}(\nu_k)|^2 \,\mathrm{Real}\left\{y_{n,l}^*(\nu)\frac{\partial}{\partial\nu}y_{n,l}(\nu)\right\} \quad (19)$$

$$= \nu_k - \mu \sum_n |y_{n,l}(\nu_k)|^2 \,\mathrm{Real}\left\{y_{n,l}^*(\nu) z_{n,l}(\nu)\right\} \quad (20)$$

where $\mathbf{z}_l = D\frac{\partial}{\partial\nu}F_\nu \mathbf{x}_l$ and μ is a step size. Therefore, one additional FFT is required in order to calculate $\mathbf{z}_l$ for update. In [12], the authors circumvent this problem by introducing the following rough approximation of $\frac{\partial}{\partial\nu}\mathbf{y}_l$

$$\frac{\partial}{\partial\nu}y_{n,l}(\nu) \approx \frac{1}{2}(y_{n,l}(\nu+1) - y_{n,l}(\nu-1)) \quad (21)$$

$$\approx \frac{1}{2}(y_{n+1,l}(\nu) - y_{n-,l}(\nu)) \quad (22)$$

using the fact that $y_{n,l}(\nu + k) = y_{n+k,l}(\nu)$ in the absence of noise. In section V it will be shown that the performance of these two updates are almost the same.

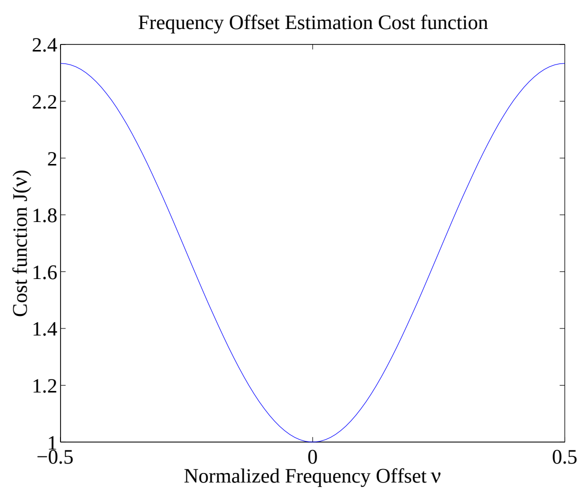

Figure 3: Frequency offset estimation cost function for ideal channel

IV. Connection To the Other Algorithms

A. ML approximations

In [11], [12] the log-likelihood function for ν under the assumption of an ideal channel ($\Lambda_k = 1$) has been approximated by

$$\Gamma(\nu) = \sum_{n=0}^{N-1} \ln(1 + \frac{1}{\sigma_w^4}|y_{n,l}(\nu)|^2) \quad (23)$$

In [12] the authors attempt to directly maximize (23), while in [11] minimization of a simpler approximation, $\sum_n |y_{n,l}(\nu)|^2$, is proposed using the first order approximation, $\ln(1+x) \approx x$. However, as Theorem 1 indicates this quantity is constant and

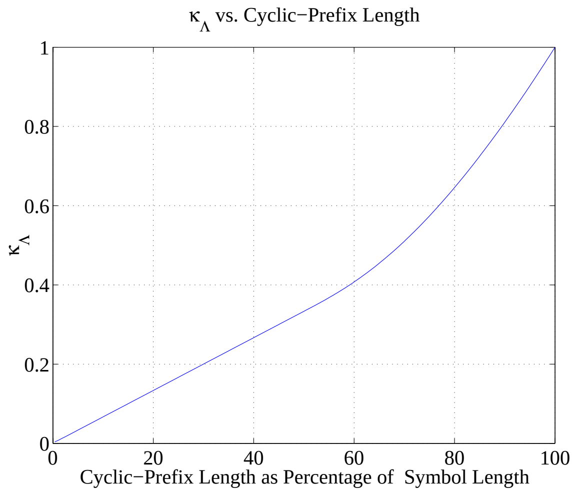

Figure 4: κ_Λ v.s. cyclic prefix length as percentage of symbol length

not a function of ν. However, depending on the implementation of an OFDM system, symbol shaping may be used, and in such a case the output energy might not be constant. But still the difference from a constant cost function would be too insignificant to be used for robust frequency offset estimation. Using a further finer approximation, $\ln(1+x) \approx x - \frac{x^2}{2}$, we have

$$\Gamma(\nu) \approx \frac{1}{\sigma_w^4} \sum_n |y_{n,l}(\nu)|^2 - \frac{1}{2\sigma_w^8} \sum_n |y_{n,l}(\nu)|^4 \tag{24}$$

From this equation, one can interpret the algorithm (20) proposed in this paper as a refined closed approximation of ML-approximation for the ideal channel case.

B. Constant Modulus Algorithm

An OFDM system can be viewed as a multi-input multi-output (MIMO) system and the distortion caused by the inaccuracy in synchronization can be modeled as a MIMO channel. In this framework, removal of ICI and ISI can be interpreted as a source separation problem. Constant Modulus Algorithm (CMA) is a popular algorithm used for blind equalization and also source separation [16], [18]. Assuming an appropriate frequency domain equalizer (FEQ), minimizing the following CM cost function will attempt to reduce ICI and ISI caused by the carrier frequency offset.

$$J(\nu) = \sum_n \beta_n E\left\{(|y_{n,l}|^2 - \gamma)^2\right\} \tag{25}$$

$$= \sum_n \beta_n E\left\{|y_{n,l}|^4\right\} - 2\gamma \sum_n \beta_n E\left\{|y_{n,l}|^2\right\} + N\gamma^2 \tag{26}$$

where $\{\beta_n\}$ are a set of weighting factors. Since $\sum_n E\{|y_{n,l}|\}^2$ is a constant, minimizing the proposed cost function (8) is equivalent to minimizing CMA for $\beta_n = 1$ in (26).

V. Simulation Results and Conclusion

Simulations have been run under the following conditions.

i) The symbols $\mathbf{s}_l$ are QPSK, i.e. $\kappa_s = 1$.

ii) An OFDM symbol consists of 64 samples.

iii) The cyclic prefix is 16 samples long (25% of symbol length).

iv) The channel has 16 paths, with random phase and exponential power delay profile, i.e.,

$$E\,|c[k]|^2 = \exp(-k/3) \tag{27}$$

v) Signal to noise ration (SNR) is 30dB.

Figure 5 confirms the global convergence of this algorithm with respect to exact gradient update considered in (20). Parameter errors corresponding to different initializations converge to zero for the ideal channel in 30dB SNR. Figure 6 compares the two updates considered in Section B for a non ideal channel in 30dB SNR. For a multipath channel without FEQ the algorithm suffers from high excess MSE and thus small step size is required. Thus, parameter trajectories in Figure 6 exhibits slow convergence. Notice that the performance difference between the exact update of (8) and the approximated update of (22) are almost negligible.

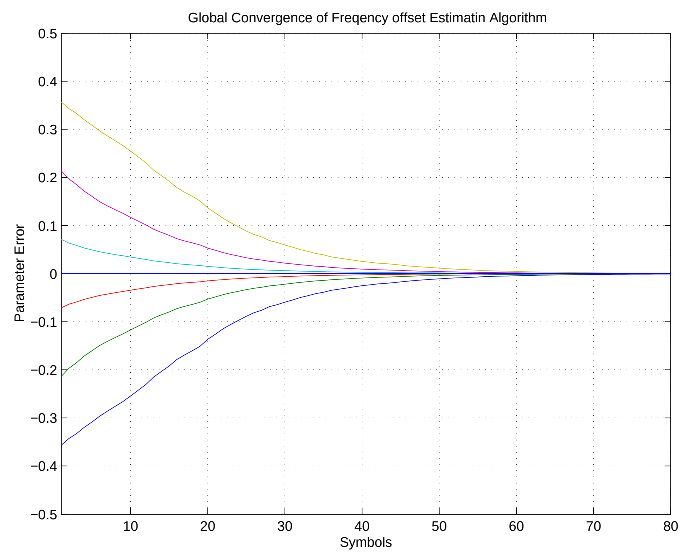

Figure 5: Global Convergence of Parameter Error

In conclusion, the proposed carrier frequency estimation algorithm based on the fourth power accumulation performs perfect carrier frequency synchronization independently from the channel and the initialization. Further developments may include performance analysis on non-idealities such as the presence Doppler shifts and the presence of channels of which length is larger than that of cyclic prefix.

VI. Appendix

A. Proof of Property 1

The property $i)$ can be proven by either direct approach or by using Parseval's Theorem [19] as done in this proof. For an DFT input $x(\nu) = [1\ e^{2\pi\nu/N} \cdots e^{2\pi(N-1)\nu/N}]$, the output power is given by $\|D\mathbf{x}(\nu)\|^2 = N \sum I_k^2(\nu)$. On the other hand, $\|\mathbf{x}(\nu)\|^2 = N$. From Parseval's Theorem, or W being a unitary matrix, we have

$$N \sum_k I_k^2(\nu) = N \tag{28}$$

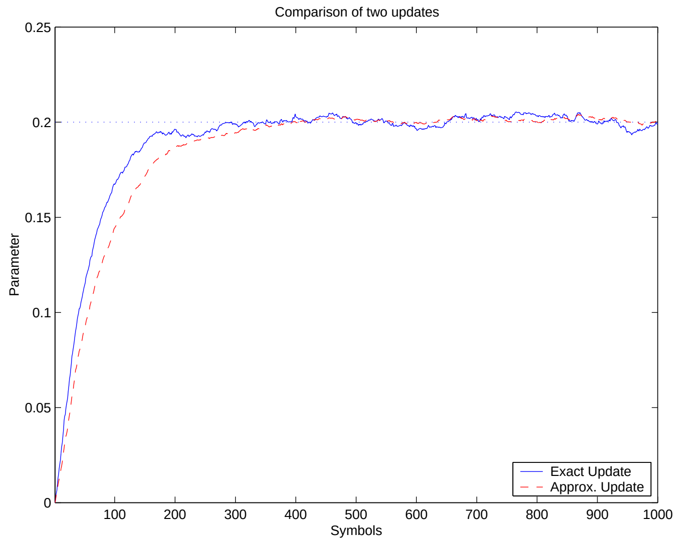

Figure 6: Comparison of exact update vs. approximated update

For the property *ii*), we assume that N is power of 2. From $\sin(\pi + x) = -\sin(x)$, we have

$$\sum_k I_k^4(\nu) = \frac{\sin^4(\pi\nu)}{N^4} \sum_{k=0}^{N-1} \frac{1}{\sin^4(\pi k/N + \pi\nu/N)} \tag{29}$$

Since N is even, we can pair the elements of $\{k/N\}_{k=0,\ldots,N-1}$ into $\{k/N,\ k/N+1/2\}_{k=0,\ldots,N/2-1}$, which leads us to

$$\sum_k I_k^4(\nu) = \frac{\sin^4(\pi\nu)}{N^4} \sum_{k=0}^{N/2-1} \frac{1}{\sin^4(\pi k/N + \pi\nu/N)} + \frac{1}{\cos^4(\pi k/N + \pi\nu/N)} \tag{30}$$

$$= \frac{\sin^4(\pi\nu)}{N^4} \sum_{k=0}^{N/2-1} 2^4 \frac{1 - 2\sin^2(\pi k/N + \pi\nu/N)}{\sin^4(\pi k/N + \pi\nu/N)}, \tag{31}$$

where we used $\sin(\pi/2 + x) = \cos(x)$ and some trigonometry identities. Successively repeating the above maneuver $\log_2 N$ times gives the final form

$$\sum_{k=0}^{N-1} I_k^4(\nu) = \frac{\sin^4(\pi\nu)}{N^4} N^4 \frac{1 - \frac{2}{3}(1 - (\frac{1}{4})^{\log_2 N})\sin^2(\pi\nu)}{\sin^4(\pi\nu)} \tag{32}$$

$$= 1 - \frac{2}{3}(1 - (\frac{1}{4})^{\log_2 N})\sin^2(\pi\nu) \tag{33}$$

■

B. Proof of Theorem 1

Let $\bar{y}_n$ denote the signal part of y_n, i.e., $y_n = \bar{y}_n + w_n$. Since the source s_n and noise w are uncorrelated, for each subchannel we have

$$E\left\{|\bar{y}_n(\nu) + w_n|^4\right\} = E|\bar{y}_n|^4 + 2\kappa_w E|\bar{y}_n|^2 \sigma_w^2 + E\left\{w_n^4\right\} \tag{34}$$

From the i.i.d. property of the source, we have

$$E|\bar{y}_n(\nu)|^2 = \sum_k \left|\Lambda_{(k-n)}\right|^2 I_k^2(\nu) \tag{35}$$

$$E|\bar{y}_n(\nu)|^4 = \kappa_s \sum_k \left|\Lambda_{(k-n)}\right|^4 I_k^4(\nu) + \kappa_w \sum_{k\neq m} \left|\Lambda_{(k-n)}\right|^2 \left|\Lambda_{(m-n)}\right|^2 I_k^2(\nu) I_m^2(\nu) \tag{36}$$

where $(k-n)$ denotes $k-n \mod N$. The cost function can now be written as

$$\sum_n E\left\{|\bar{y}_n(\nu) + w_n|^4\right\} = \sum_n E\left\{|\bar{y}_n|^4\right\} + \sigma_w^2 \sum_n E\left\{|\bar{y}_n(\nu)|^2\right\} + N\kappa_w\sigma_w^2 \tag{37}$$

Notice that

$$\sum_n E\left\{|\bar{y}_n(\nu)|^2\right\} = \sum_k |\Lambda_k| \sum_k I_k(\nu)^2 = \|\Lambda\|^2, \tag{38}$$

which proves part *i*) and

$$\sum_n E|\bar{y}_n(\nu)|^4 = \kappa_s \sum_k |\Lambda_k|^4 \sum_k I(k)^4 + \kappa_w \sum_{k\neq m} |\Lambda_k|^2 |\Lambda_m|^2 \sum_{k\neq m} I_k^2(\nu) I_m^2(\nu)$$

$$= \left(\kappa_s \sum_k |\Lambda_k|^4 - \kappa_w \sum_{k\neq m} |\Lambda_k|^2 |\Lambda_m|^2\right) \sum_k I(k)^4 + \kappa_w \sum_{k\neq m} |\Lambda_k|^2 |\Lambda_m|^2 \tag{39}$$

$$= \|\Lambda\|^4 \left((\kappa_s + \kappa_w)\kappa_\Lambda - \kappa_w\right) \sum_k I(k)^4 + \kappa_w \sum_{k\neq m} |\Lambda_k|^2 |\Lambda_m|^2$$

$$= \|\Lambda\|^4 \left((\kappa_s + \kappa_w)\kappa_\Lambda - \kappa_w\right) \sum_k I(k)^4 + \kappa_w \|\Lambda\|^4 (1 - \kappa_\Lambda) \tag{40}$$

where, we have used the relation

$$1 = \left(\sum_k I_k^2(\nu)\right)^2 = \sum_k I_k^4(\nu) + \sum_{k\neq m} I_k^2(\nu) I_m^2(\nu) \tag{41}$$

Combining (40) and Property 1-*ii*) proves part *ii*). ■

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