2000 Conference on Information Sciences and Systems, Princeton University, March 15-17, 2000

# A BLIND ADAPTIVE EQUALIZER FOR FSK SYSTEMS BASED ON A CHARACTERIZATION OF MULITPATH ON THE FSK SIGNALS

Wonzoo Chung*, Richard K. Martin*, Michael Ready†, and C. Richard Johnson Jr.*
*School of Elec. Eng.
Cornell University
Ithaca, NY 14853
†Applied Signal Technology
Sunnyvale, CA 94086

*Abstract* — **This paper investigates a blind equalization scheme for the continuous phase modulated frequency shift keying (CPFSK) system that appeared in [4], based on a characterization of multipath on the FSK signals. The distortion on the phase demodulated FSK signals due to a multipath channel can be modeled as a linear time varying filtering. The time variation of the linear filter model is slow for a mild multipath channel and fast for a severe mulitpath channel. Therefore, the constant modulus algorithm (CMA), which is the most practical and popular blind equalization algorithm, is applied on the pre-demodulated signals in order to mitigate the severeness of the multipath channel. Since CMA cannot achieve perfect equalization of pre-demodulated signals due to the correlated nature of CPFSK signals, the demodulated signals still suffer from the distortion due to a relatively slow time varying filter. Thus, a decision directed LMS equalizer is deployed to the de-modulated signal in order to track and mitigate the distortion due to the time varying filtering.**

# I Introduction

Modern digital communication systems utilize various digital signal processing techniques in order to enhance receiver performance. Digital filtering for channel equalization is, perhaps, the most famous and successful example where an adaptive filter mitigates inter symbol interference of linearly modulated signals such as QAM. For linearly modulated signals the distortion due to a multipath channel is well modeled by linear filtering; hence, equalization is basically achieved by deconvolution of the baud spaced or fractionally (usually $T/2$) spaced sampled received signal [5].

However, for non-linear modulation schemes such as FSK, the effect of a multipath channel is modeled not as linear filtering but as non-linear distortion. Still, adaptive filtering can serve as an effective means of mitigating the non-linear distortion. The non-linear distortion can be viewed as a time varying linear filtering operation. Since the functions describing the time variation of the coefficients are quite complicated, equalization of this non-linear distortion is considered difficult. Instead of indirect equalization (based on estimating the time variation and designing an equalizer to counteract this time variation), a blind adaptive equalization algorithm, such as decision directed LMS (DD-LMS), can be successfully used for equalizing the time varying linear filter model of the non-linear distortion, when the time variation is reasonably slow. The speed of the time variation is related to the severeness of the multipath channel: for a mild multipath channel the time variation is slow, while it is fast for a severe multipath channel. Therefore, for a severe multipath channel, a blind adaptive algorithm such as CMA [6] can be applied on the FSK signal (pre-Demodulation) in order to reduce the severeness of the multipath channel.

In section 2, we review the general continuous phase FSK (CPFSK) scheme and characterize the non-linear distortion of a CPFSK receiver due to a multipath channel. The relation between the speed of time variation of the distortion and the severity of a multipath channel is illustrated with a two tap multipath channel model. In section 3, we apply the blind adaptive pre-demodulation filter and post-demodulation filter on the FSK signals. Simulation results on FSK systems as well as the GSM system are presented

[1]This work was supported in part by NSF Grant ECS-9811297 and Applied Signal Technology.

# II Multipath Distortion of CPFSK systems

## A Review of a CPFSK system and notations

We consider a CPFSK scheme starting with a discrete PAM symbol set [3].

$$s_n \in \{\pm 1, \pm 3, ..., \pm(M-1)\}$$

The discrete symbol sequence $[..., s_0, s_1, ..., s_n ...]$ is converted to a continuous time signal $v(t)$ by a pulse shaping filter $g_T$ with baud interval $T$.

$$v(t) = \sum s_n g_T(t - nT), \tag{1}$$

In order to obtain as smooth phase signal as possible, $v(t)$ is integrated and then frequency modulated. The base band representation of the transmitter output is

$$\psi(t) := \int_{-\infty}^{t} v(\tau) d\tau \tag{2}$$
$$u(t) = \exp[j\beta\psi(t) + j\zeta_0], \tag{3}$$

where $\beta$ is a constant and we assume the phase offset $\zeta_0 = 0$ henceforth.

Assuming a linear channel $\mathbf{c}$, the received signal is represented as

$$r(t) = \mathbf{c}(t) \star \mathbf{u}(t) + w(t), \tag{4}$$

where $w(t)$ is the white Gaussian noise process, and $\star$ denotes convolution. Suppose that $c(t)$ is a finite time support multipath channel with

$$\mathbf{c}(t) = \sum_{i=0}^{i=N_c} c_i \delta(t - it_0) \tag{5}$$

where $t_0$ denotes the time delay of the multipath and $c_i$ denotes the complex valued multipath coefficient of which magnitude is $\rho_i$ and phase is $\phi_i$, i.e.

$$c_i = \rho_i e^{j\phi_i}. \tag{6}$$

In the receiver, the received signal $r(t)$ is demodulated by extracting an estimation of $\psi(t)$ as the phase of $r(t)$ and differentiated.

$$\hat{\psi}(t) = \angle r(t) \tag{7}$$

$$\hat{v}(t) = \frac{d}{dt}\hat{\psi}(t) \tag{8}$$

Finally, the output of the receiver is given by the matched filter output with respect to $\hat{v}(t)$.

$$\hat{s}_n = \hat{v}(t) \star g_T^*(-t)|_{t=nT} \tag{9}$$

## B Time Varying Filtering Model of Multipath Distortion

In this section, we develop a mathematical model of the nonlinear distortion due to the multipath channel on the frequency discriminated FSK signals. The nonlinear distortion is modeled as a time varying linear filter of the same length as that of the channel causing distortion. The rate of time variations depends on the severity of the multipath channel.

Henceforth, we assume an absence of channel noise, and consider only the effect of multipath channel. Without loss of generality, we further assume $\beta = 1$. We are establishing the relation between the pulse shaped source signal $v(t)$ in (1) and the demodulated received signal $\hat{v}(t)$ in (8).

$$r(t) = \sum_{i=0}^{N_c} c_i u(t - it_0) = \sum_{i=0}^{N_c} \rho_i e^{j(\psi(t-it_0)+\phi_i)} \tag{10}$$

Define

$$r_X(t) := \sum \rho_i \cos(\psi(t - it_0) + \phi_i) \tag{11}$$

$$r_Y(t) := \sum \rho_i \sin(\psi(t - it_0) + \phi_i) \tag{12}$$

Then

$$r(t) = \sqrt{r_X(t)^2 + r_Y(t)^2} \exp\left\{ j \tan^{-1}\left(\frac{r_Y(t)}{r_X(t)}\right)\right\} \tag{13}$$

Therefore, from (7)

$$\hat{\psi}(t) = \tan^{-1}\left(\frac{r_Y(t)}{r_X(t)}\right) \tag{14}$$

and from (8)

$$\hat{v}(t) = \frac{d}{dt}\tan^{-1}\left(\frac{r_Y(t)}{r_X(t)}\right) \tag{15}$$

$$= \frac{1}{\|r(t)\|^2}\left(r_X(t)\dot{r}_Y(t) - r_Y(t)\dot{r}_X(t)\right) \tag{16}$$

where we denote $\dot{r}_X(t) := \frac{d}{dt} r_X(t)$. Since

$$\dot{r}_X(t) = -\sum \rho_i \sin(\psi(t - it_0) + \phi_i) v(t - it_0) \tag{17}$$

$$\dot{r}_Y(t) = \sum \rho_i \cos(\psi(t - it_0) + \phi_i) v(t - it_0), \tag{18}$$

by defining

$$\begin{aligned} g_i(t) := \; & \frac{\rho_i}{\|r(t)\|^2}(r_X(t)\cos(\psi(t - it_0) + \phi_i) \\ & + r_Y(t)\sin(\psi(t - it_0) + \phi_i)) \end{aligned} \tag{19}$$

and a time varying linear filter,

$$\mathbf{g}(t) := \sum_{i=0}^{N_c} g_i(t)\delta(t - it_0) \tag{20}$$

one can check $\hat{v}(t)$ can be expressed as

$$\hat{v}(t) = \mathbf{g}(t) \star v(t) \tag{21}$$

Notice that $g_i(t)$ is a distorted version of $\|c_i\|$. (Recall that $\rho_i$ and $\phi_i$ are, respectively, the magnitude and phase of $c_i$. Note that $\rho_i$ and $\phi_i$ appear explicitly in the definition of $g_i$ in (19), and implicitly via $r_X$ and $r_Y$ in (11) and (12).) Now we have a time varying linear model for the multipath distortion of frequency modulation schemes. The multipath distortion is now characterized by the behavior of $\mathbf{g}(t)$. Some very interesting questions about $\mathbf{g}(t)$ from an adaptive equalization viewpoint would be

- Is $\mathbf{g}(t)$ always non-zero and bounded ?
- What is the main factor determining the speed of time variation of $\mathbf{g}(t)$ ?

We examine two properties of $\mathbf{g}(t)$, which help to answer these two questions.

**Property 1** The time varying linear filter model $\mathbf{g}(t)$ for mulipath distortion of FSK has the following properties:

i) $\sum g_i(t) = 1$ as long as $\|r(t)\| \neq 0$. Thus, $\mathbf{g}(t) \neq 0$, unless $\mathbf{c}(t) = 0$.

ii) Denote

$$\varphi_{ik}(t) := \psi(t - it_0) - \psi(t - kt_0) + \phi_i - \phi_k, \tag{22}$$

then $\mathbf{g}(t)$ is a rational function of linear combinations of

$$\{ \cos(\varphi_{ik}(t)) \}_{i,k=1,\ldots,N_c}$$

**Proof:** The proof of i) follows from using (11) and (12) in (19) to reveal

$$\begin{aligned}\sum_i g_i(t) &= \frac{1}{\|r(t)\|^2}\Big(r_X(t)\sum \rho_i \cos(\psi(t-it_0)+\phi_i)\\ &\quad + r_Y(t)\sum_i \rho_i \sin(\psi(t-it_0)+\phi_i)\Big)\\ &= \frac{1}{\|r(t)\|^2}(r_X(t)r_X(t)+r_Y(t)r_Y(t)) \qquad (23)\\ &= 1 \qquad (24)\end{aligned}$$

as long as $\|r(t)\| \neq 0$. Since $\sum g_i(t) = 0$ is a necessary condition for $\mathbf{g}(t) = 0$, $\mathbf{g}(t) \neq 0$, unless $\mathbf{c}(t) = 0$. The proof of ii) follows from trigonometric identities. Let

$$g_i = \frac{1}{\|r(t)\|^2}\alpha_i \qquad (25)$$

where

$$\begin{aligned}\alpha_i :=\ & \rho_i(r_X(t)\cos(\psi(t-it_0)+\phi_i)\\ & + r_Y(t)\sin(\psi(t-it_0)+\phi_i))\\ =& \sum_k \rho_i\{\rho_k\cos(\psi(t-kt_0)+\phi_k)\cos(\psi(t-it_0)+\phi_i)\\ & + \rho_k \sin(\psi(t-kt_0)+\phi_k)\sin(\psi(t-it_0)+\phi_i)\}\\ =& \sum_k \rho_i\rho_k\cos(\psi(t-kt_0)-\psi(t-it_0)+\phi_k-\phi_i)\end{aligned}$$

Since $\|r(t)\|^2 = \sum \alpha_i$,

$$g_i = \rho_i \frac{\sum_k \rho_k \cos(\varphi_{ik}(t))}{\sum_i\sum_k \rho_i\rho_k\cos(\varphi_{ik}(t))}. \qquad (26)$$

Although $\sum g_i(t) = 1$, it is possible that individual $|g_i(t)| \to \infty$ when $\|r(t)\| \to 0$ via destructive interference. However, such destructive interference rarely occurs for non-fading channels. Therefore, we assume $\|r(t)\|^2 > 0$ and $g_i(t)$ are reasonably bounded values.

Since $\mathbf{g}(t)$ is a function of $\{\cos(\varphi_{ik}(t))\}$, the time variation of $\mathbf{g}(t)$ is determined by the time variation of $\{\cos(\varphi_{ik}(t))\}$ and their contribution to $\mathbf{g}(t)$ through $\{\rho_i\rho_k\}$. On the other hand, we have

$$\varphi_{ik}(t) = \int_{\tau=t-it_0}^{\tau=t-kt_0} v(\tau)d\tau + \phi_i - \phi_k \qquad (27)$$

Since $v(\tau)$ is generated by an i.i.d. source sequence $\{s_n\}$, $\cos(\varphi_{ik}(t))$ is also observed to behave like a random source independent from $t_0$ when $t_0 > T$ (recall that $T$ is the baud interval). Thus, the main factor which determines time variation of $\mathbf{g}(t)$ is the scale factor $\rho_i\rho_k$ determing the contribution of the random variable $\cos(\varphi_{ik}(t))$ to $\mathbf{g}(t)$.

¿From (26), for a channel satisfying a condition that the amplitude of the cursor dominates over other spikes, explicitly, the amplitude of the cursor is greater than the sum of the amplitude of spikes other than the cursor, we can obtain an approximation of the bound of $g_i$ which reveals the relation between the dynamic range of $g_i$ and the severity of a multipath channel. For a such channel, it is observed that the amplitude of the cursor, say $\rho_{i_c}$, has the maximum variance among other taps, as (26) suggests. Thus, we consider a bound of $g_{i_c}(t)$ as a representative of $\mathbf{g}(t)$. In (26)

$$\begin{aligned}\sum_k \rho_k^2 - \sum_{k\neq l}\rho_k\rho_l &\leq \sum_i\sum_k \rho_i\rho_k\cos(\varphi_{ik}(t))\\ &\leq \sum_k \rho_k^2 + \sum_{k\neq l}\rho_k\rho_l \qquad (28)\end{aligned}$$

We further assume that $\rho_{i_c}$ so dominates other $\rho_i$'s that $\rho_i\rho_k$ is ignorable for $i, k \neq i_c$, then

$$\begin{aligned}(\rho_{i_c} - \sum_{k\neq i_c}\rho_k)^2 &= \sum \rho_k^2 - 2\rho_{i_c}\sum\rho_k + \sum_{i\neq k, i, k\neq i_c}\rho_i\rho_k\\ &\approx \sum \rho_k^2 - 2\rho_{i_c}\sum\rho_k - \sum_{i\neq k, i, k\neq i_c}\rho_i\rho_k\\ &= \sum_k \rho_k^2 - \sum_{k\neq l}\rho_k\rho_l \qquad (29)\end{aligned}$$

Finally, we have

$$\frac{\rho_{i_c} - \sum_{k\neq i_c}\rho_k}{(\rho_{i_c} + \sum_{k\neq i_c}\rho_k)^2} \leq g_{i_c}(t) \leq \frac{\rho_{i_c} + \sum_{k\neq i_c}\rho_k}{(\rho_{i_c} - \sum_{k\neq i_c}\rho_k)^2} \qquad (30)$$

¿From (30), we can observe that as a channel becomes severe ($\rho_{i_c} - \sum_{k\neq i_c}\rho_k \to 0$), the upperbound of $g_i$ dramatically increases. Therefore, we conclude:

**Property 2** A severe multipath channel tends to have more rapid time variation of $g(t)$ then a mild multipath channel regardless of the delay length.

## C Example: Two tap channel case

Encouraged by the observation that a variety of multipath channels are well approximated by a channel model with a few taps, we closely analyze a simple non-trivial channel model as an example. This simple channel model leads to the most important consequences of multipath distortion observed in the preceding section.

Let a multipath channel $\mathbf{c}$ be given by

$$\mathbf{c}(t) = \delta(t) + \rho e^{j\phi}\delta(t-t_0) \qquad (31)$$

Suppose $\beta = 1$ and define

$$\varphi(t) = \psi(t) - \psi(t-t_0) - \phi \qquad (32)$$

Straightforward calculations yield

$$\begin{aligned}\alpha_0 &= 1 + \rho\cos(\varphi(t)) \qquad (33)\\ \alpha_1 &= \rho^2 + \rho\cos(\varphi(t)) \qquad (34)\\ \|r(t)\|^2 &= \sum_i \alpha_i = 1 + 2\rho\cos(\varphi(t)) + \rho^2 \qquad (35)\end{aligned}$$

$$\mathbf{g}(t) = [\frac{1+\rho\cos(\varphi(t))}{1+2\rho\cos(\varphi(t))+\rho^2}\ \frac{\rho^2+\rho\cos(\varphi(t))}{1+2\rho\cos(\varphi(t))+\rho^2}] \qquad (36)$$

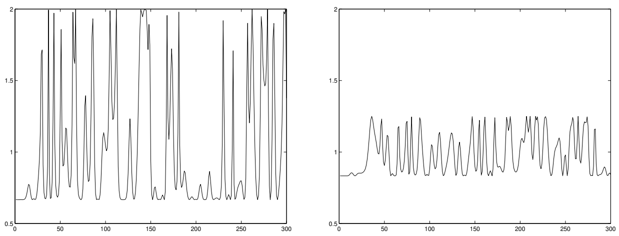
Notice that a destructive interference occurs when the two taps have same magnitude with opposite phase ($\rho = 1$ and $\varphi = \pi$), and $|g_0| \to \infty$ and $|g_1| \to \infty$ in such case. In Figure 1, we compare the time variation of $g_0$ for a severe multipath ($\rho = 0.5$) and a mild multipath ($\rho = 0.2$) when $\phi = 0$. (Recall that $g_1 = 1 - g_0$.) This simple two tap channel model yields a tighter bound of $g_0(t)$ than (30);

$$\frac{1}{1+\rho} \le g_0(t) \le \frac{1}{1-\rho} \tag{37}$$

which yields

$$\begin{array}{ll} 0.67 \le g_0 \le 2 & \text{for } \rho = 0.5 \\ 0.83 \le g_0 \le 1.25 & \text{for } \rho = 0.2, \end{array}$$

as confirmed in Figure 1.



a) Severe Case $\rho = 0.5$ b) Mild Case $\rho = 0.2$

Figure 1: Time variation of $g_0$

# III Equalization

## A Blind Adaptive Post-D and Pre-D filter

Figure 2 and 3 illustrate blind adaptive equalization schemes for CPFSK appeared in [4]. Figure 2 shows a equalizer using DD-LMS, which is referred to as a post-D filter. Notice that as a result of the time-varying channel model (21) the length constraint of the post-D filter is the same as that of conventional T/2 linear equalizer for the channel on the FSK signal. For severe multipath channels, in order to deal with fast time

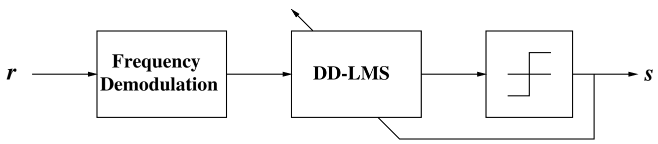



Figure 2: Post-D filter

variation of $\mathbf{g}(t)$, CMA equalization can be applied for the received FM signal before demodulation (which is referred to as a pre-D filtering). However, CMA requires an i.i.d. source for successful equalization [1], while the transmitter output $\{u(t)|_{t=nT_S}\}$ is not an i.i.d. source in general (since it is generated through an integration operator). Nonetheless, we can reasonably well approximate the FM transmitter output $\{u(t)|_{t=nT_S}\}$ as an i.i.d. source for a reasonable modulation index $\beta$. Although pre-D CMA filter may not achieve perfect equalization, it mitigates severe multipath channel so that post-D DD-LMS algorithm can equalize $\mathbf{g}(t)$.

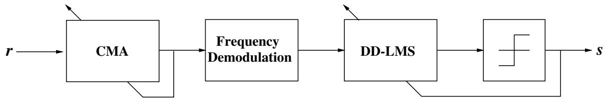



Figure 3: Pre-D filter & Post-D filter

## B Simulation Results

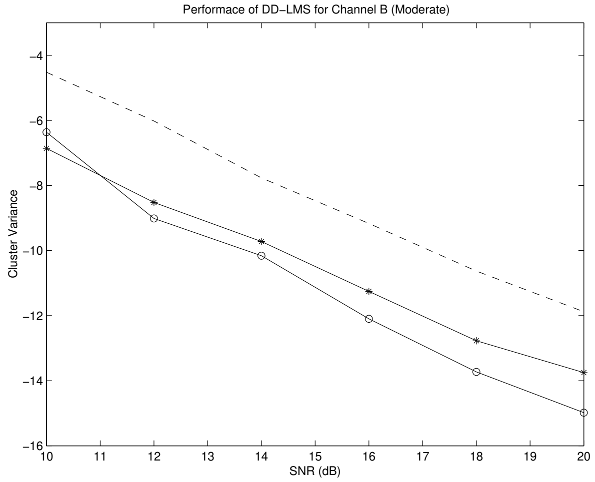
### B.1 The effect of DD-LMS for two tap channels

Let us examine the performance of DD-LMS for the two tap channel model considered in section 2.3. Channel A and B are two tap channel with the delay $t_0 = 3T$ (3 baud intervals) and $\rho = 0.5$ and $\rho = 0.2$ respectively. We used raised cosine pulse shaping with 4 samples per baud interval for 2-level FSK. In the absence of noise, the variance of error due to multipath distortion is $-19$dB and $-14$dB respectively. Figure 4 illustrates the performance gain of the T/2 post-D DD-LMS filter of length 10 in the presence of noise. We can observe about 3dB gain due to the post-D filter for both channels. A larger step size is observed to enhance the performance of the post-D filter for both channels at high SNR (since a large step size enhances the tracking ability of the filter). Especially for the severe channel A the enhancement is more remarkable for high SNR.

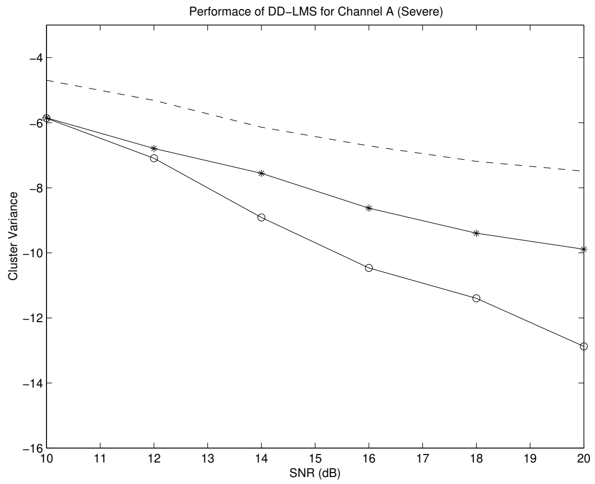
### B.2 Simulation results for GSM system

The Global System for Moblie Communication (GSM), one of most widely used wireless cellular communication systems, uses Gaussian Mean Shift Keying (GMSK), which is a specific type of CPFSK. We apply the proposed methods for this system as an example. Since GMSK for the GSM system has very little excess bandwith ([7]), this system is not an ideal example for which the proposed method exhibits its full potential. However, it still exhibits reasonable performance improvement. We have used $T/8$-sampling for pre-demodulated signals and 64 feedforward CMA taps (which span 8 baud samples) as the pre-D filter and baud spaced DD-LMS and DFE (4 feed forward filter and 7 feed back filter). The channel we used for the simulation results in Figure 5 is a synthetic channel $c = [0.1{+}0.2i,\ 1,\ 0.1{+}0.2i,\ 0.6{-}0.25i,\ {-}0.2{+}0.1i]$ in baud space.

Figure 5 shows bit error rate performance of three different methods. First, the dotted line shows the performance of conventional GMSK with a frequency discriminator detector with one-tap DFE (due to inherent ISI caused by the pulse shape filter). The second line presents the performance of Pre-D CMA only with frequency discriminator detector with one-tap DFE. Finally, the third line, which outperforms all of the above, shows the performance of Pre-D CMA and Post-D DD-LMS with DFE. This short simulation suggests that even in this harsh situation (very little excess bandwidth), the proposed methods present reasonable BER improvement.




A) For $\rho = 0.2$ (moderate)




B) For $\rho = 0.5$ (severe)

Dotted line : Without Adaptation
Solid line with * : stepsize=$1 \times 10^{-3}$
Solid line with o : stepsize=$2 \times 10^{-2}$

Figure 4: Performance of Post-D DD-LMS

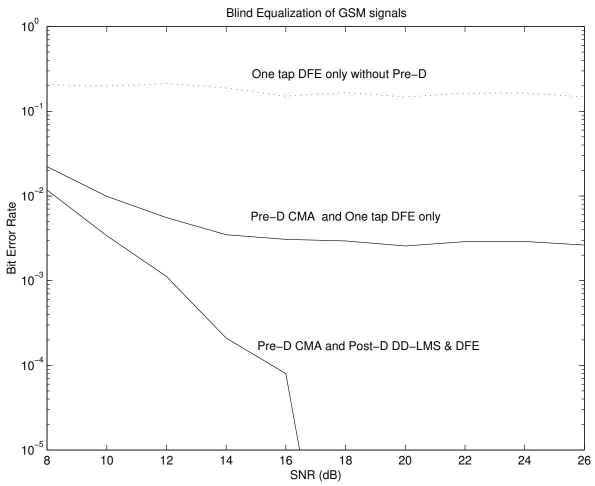



Figure 5: Blind Equalization of GSM signals

# References


[1] C. R. Johnson, Jr., P. .B. Schniter, T. S. Endres, J. M. Behm, D. R. Brown, and R. A. Casas "Blind Equalization Using the Constant Modulus Criterion: A Review" *Proc. IEEE* vol.86 no. 10, pp 1927-1950, Oct. 1998

[2] Philip F. Panter, "*Modulation, Noise, and Spectral Analysis*," McGraw-Hill Book Company, New York, 1965.

[3] John G. Proakis and Masoud Salehi, "*Communication Systems Engineering*," Prentice Hall, 1994.

[4] Michael J. Ready and Jeff Harp, "Performance Improvements Achieved by Equalizing Intermediate Rate FSK signals," *Asilomar Conference*, Pacific Grove, CA, Nov, pp.379-383, 1998.

[5] L. Tong, et al. "A New Approach to Blind Identification and Equalization of Multi-path Channels," *IEEE Trans. on Info. Theory*, IT-40:340-349, March 1994.

[6] J.R. Treichler and B. G. Agee, "A New Approach to Multipath Correction of Constant Modulus Signals", *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. ASSP-31, no. 2, pp. 459-472, April 1983

[7] European Telecommunications Standards Institute, "European digital celluar telecommunications system: GSM 05.04" Tech. Rep., ETSI. Sophia Antipolis, France, 1994.