BLIND ADAPTIVE PHASE OFFSET CORRECTION

Wonzoo Chung, William A. Sethares†, and C. Richard Johnson Jr.**

Correspondence: wonzoo@ee.cornell.edu

* School of Elec. Eng.
Cornell University
Ithaca, NY

† School of ECE
University of Wisconsin
Madison, WI

ABSTRACT

This paper introduces a blind adaptive carrier phase offset recovery scheme based on the Constant Modulus (CM) Algorithm and analyzes its behavior. The proposed CM-Derotator (CMD) minimizes the dispersion of the real projection of the derotated signal to remove the phase offset of the signal. By analyzing the cost function, the performance of the CMD is investigated in various situations including its tracking ability, its behavior in the presence of Inter Symbol Interference, and when there are statistical dependencies between the in-phase and quadrature components.

1. INTRODUCTION

In modern communication systems, the demand for transmitting increasing information over band-limited channels often prefers blind equalization over equalization schemes based on a training sequence. However, in most blind equalization schemes, the received signal suffers from a phase offset [4] caused by the distortion of the band-limited channel, or by a carrier phase error. This phase offset decreases the efficiency of the equalization since it may cause incorrect decisions to be made for two dimensional signals, or it may distort the amplitude of one dimensional signals such as in single sideband modulation. An adaptive approach to recover phase offset was proposed and analyzed in [3] using a decision directed MSE style algorithm. This paper proposes a blind adaptive scheme called the Constant Modulus Derotator (CMD) for removing this phase offset, which is based on the CMA algorithm of [5]. Two applications are detailed: the use of CMD as a pre-equalization module for real (PAM) signal constellations such as the HDTV 8-VSB standard, and the use of the CMD as a post-equalization module for complex (QAM) constellations. The latter is exploited as a key ingredient in a constellation identification procedure. Thus the CMD can be used either before or after equalization depending on the particular system requirements.

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Section 2 derives the CMD algorithm, and then analyzes its local minima and maxima by studying the cost function. In section 3, the characteristics of the CMD are specified for a number of signal constellations. Section 4 investigates the tracking ability of the CMD under the assumption that the phase offset varies linearly over time. Finally in section 5, the behavior of the CMD is investigated in less ideal situations; in the presence of inter symbol interference and when there is a statistical dependence between the in-phase and quadrature components.

2. CONSTANT MODULUS DEROTATOR

2.1. Problem Description

Suppose that a complex random sequence s drawn from a finite constellation with known statistical properties suffers from an unknown constant phase offset Φ in the presence of white complex Gaussian noise w. Only the output

$$y = se^{i\Phi} + w \tag{1}$$

is measured. In order to estimate Φ and directly remove this offset, consider the single tap derotator shown in Figure 1, where ϕ represents an estimate of Φ, and the

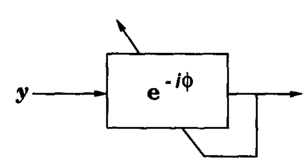

Figure 1: A Single Tap Derotator

arrow represents a way of (iteratively) updating ϕ. This can be viewed as the problem of equalizing a scalar channel, and conventional blind estimation techniques can be

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used for blind removal of Φ. However, direct application of the CM-algorithm, the most practical and popular blind algorithm [2, 5], is not feasible because of its phase invariance [2]. The following sections derive the CMD, a CM-based algorithm for the blind estimation of Φ, and analyze its behavior in a variety of situations.

2.2. Proposed Algorithm

In the absence of noise, and if ϕ were exactly equal to Φ, then the projection of y onto the real axis would consist of a collection of points at the (real part of) the symbol values defined by the constellation from which s is drawn. In the presence of noise, the projection will consist of a number of clusters centered at these symbol values. Similarly, when ϕ is offset somewhat from Φ, the clusters widen. Thus, a sensible criterion for estimating Φ is to try and minimize the dispersion of the projection of the constellation onto the real axis. Formally, consider the cost function

$$J(\phi) = E\left\{(\Re(ye^{-i\phi})^2 - \gamma)^2\right\} \qquad (2)$$

where γ is any real constant (possibly zero) and $\Re(\cdot)$ denotes the real projection operator (i.e. $\Re(a+bi) = a$). For many signal constellations, the ϕ that minimizes $J(\phi)$ will be equal to Φ, but for some signal constellations the dispersion can be made even smaller by projecting onto a line other than the real axis. For example, section 4.2 shows that the V29 signal constellation minimizes $J(\phi)$ when $\phi = \Phi + \frac{\pi}{4}$.

Using a stochastic gradient algorithm to minimize (2) gives the *Constant Modulus Derotator* (CMD) algorithm

$$\phi_{k+1} = \phi_k - \mu\left(\Re(ye^{-i\phi_k})^2 - \gamma\right)\Re(ye^{-i\phi_k})\,\Im(ye^{-i\phi_k}) \qquad (3)$$

which uses the fact that $\frac{\partial}{\partial\phi}\Re(ye^{-i\phi_k}) = \Im(ye^{-i\phi_k})$, where $\Im(\cdot)$ denotes projection onto the imaginary axis, i.e., $\Im(a+bi) = b$.

2.3. Analysis of the Cost Function

One of the most revealing ways to understand the behavior of an algorithm such as (3) is to study the cost function to examine the "error surface" over which the algorithm evolves. Some assumptions on the statistical properties of the sequence $\{s\}$ are required to make this concrete.

Assumptions: Let s_R and s_I denote the Real (in-phase) part and the Imaginary (quadrature) part of the sequence s respectively.

i) $\{s_R\}$ and $\{s_I\}$ are sub-Gaussian, i.e. their kurtosis, κ_{s_R} and κ_{s_I}, are less than 3. (Recall that the kurtosis of a random sequence s is defined as $\kappa_s := E\{s^4\}/E\{s^2\}^2$)

ii) (a) $E\{s_R^2 s_I^2\} = E\{s_R^2\}E\{s_I^2\}$, and (b) $E\{s_R^l s_I^m\} = 0$ for all integers l and m with $l+m \le 3$ (except $l=m=2$).

iii) $E\{s_R^2\} = E\{s_I^2\}$.

iv) w is a white circular complex Gaussian noise that is independent from the sequence s.

We will later consider cases where some of conditions in ii)-iii) are violated. For convenience, let $m_2 := E\{s_R^2\} = E\{s_I^2\}$, $m_{R4} := E\{s_R^4\}$ and $m_{I4} := E\{s_I^4\}$. Let $\theta := \Phi - \phi$ represent the parameter error of the CMD. Without loss of generality, the cost function (2) can be described in terms of θ.

Property 1 (Stationary Points of the CMD) *Under assumptions i) through iv), the CMD has the following stationary points:*

Local Minima: $\theta = 0,\ \pi/2,\ \pi,\ 3\pi/2,$ *and* 2π.

Local Maxima: $\theta = \pm\sin^{-1}\sqrt{(3-\kappa_{s_R})/(6-\kappa_{s_R}-\kappa_{s_I})}$.

Proof: The cost function (2) can be rewritten as

$$J(\theta) = E\left\{(\Re(se^{i\theta} + \omega e^{-i\phi})^2 - \gamma)^2\right\}$$

Because ω is circular Gaussian, its statistics are the same as those of $\omega e^{-i\phi}$. Hence

$$J(\theta) = m_{R4}\cos^4\theta + m_{I4}\sin^4\theta + \frac{3m_2^2}{2}\sin^2 2\theta - 2\gamma(m_2+\sigma^2) + \sigma_4 + \gamma^2$$

where $\sigma^2 = E\{w_R^2\}$ and $\sigma_4 = E\{w_R^4\}$. The derivative of J is

$$\frac{\partial J}{\partial\theta} = 2m_2^2\sin 2\theta\{(\kappa_{s_R}+\kappa_{s_I}-6)\sin^2\theta + 3 - \kappa_{s_R}\}.$$

Since $\kappa_{s_R} < 3$, $\kappa_{s_I} < 3$, the stationary points of J occur at θ for which $\sin 2\theta = 0$ and $\sin\theta = \pm\sqrt{(3-\kappa_{s_R})/(6-\kappa_{s_R}-\kappa_{s_I})}$. The second derivative of J is

$$\frac{\partial^2 J}{\partial\theta^2} = 4m_2^2\cos 2\theta\left\{(\kappa_{s_R}+\kappa_{s_I}-6)\sin^2\theta + 3 - \kappa_s\right\} + 2m_2^2(\kappa_{s_R}+\kappa_{s_I}-6)\sin^2 2\theta,$$

and it is easy to see that $\{\theta \mid \sin 2\theta = 0\}$ are minima while $\left\{\theta \mid \pm\sqrt{(3-\kappa_{s_R})/(6-\kappa_{s_R}-\kappa_{s_I})}\right\}$ are maxima. ■

Notice that as $\kappa_{s_R}, \kappa_{s_I} \to 3$, $\frac{\partial J}{\partial\theta} \to 0$, and thus the performance may degrade as the kurtosis approaches 3.

2.4. CMD Without Norm Constraint

The CMD, by adjusting only the value of ϕ in $e^{i\phi}$ essentially constrains the magnitude of the tap to unity. This causes the update term to require the calculation of a sinusoid, which may involve some hardware complexity. This constraint can be relaxed by using a single tap complex CMA update with the cost function

$$J(f) = E\left\{(\Re(yf^*)^2 - \gamma)^2\right\}, \tag{4}$$

leading to the stochastic descent algorithm

$$\mathrm{f}_{k+1} = \mathrm{f}_k + \mu\ (\Re(y\mathrm{f}_k^*)^2 - \gamma)\ \Re(y\mathrm{f}_k^*)y. \tag{5}$$

This algorithm can be viewed as a one-tap implementation of the "modified constant modulus algorithm" independently proposed in [1]. The "constant" γ must now be adjusted to keep the power gain of the derotator at unity. Since the gradient system is the same whether represented in Cartesian or polar coordinates, let $\mathrm{f} = re^{i\theta}$. The cost can be analyzed as

$$\begin{aligned}
J(f) &= m_{R4}r^4\cos^4\theta + m_{I4}r^4\sin^4\theta + \frac{3m_2^2r^4}{2}\sin^2 2\theta \\
&\quad +3r^4\sigma^2 - 2r^2\gamma(m_2+\sigma^2) + \gamma^2 \\
\frac{\partial J}{\partial\theta} &= 2m_2^2r^4\sin 2\theta\left\{(\kappa_{s_R} + \kappa_{s_I} - 6)\sin^2\theta + 3 - \kappa_{s_R}\right\} \\
\frac{\partial J}{\partial r} &= 4r[r^2m_2^2\left(\kappa_{s_R}\cos^4\theta + \kappa_{s_I}\sin^4\theta + \frac{3}{2}\sin^2 2\theta\right. \\
&\quad \left. +3\,\frac{\sigma^2}{m_2^2}\right) - \gamma(m_2+\sigma^2)].
\end{aligned}$$

In the radial direction, there is a local maximum at $r = 0$ and a minima surface at

$$r^2 = \frac{\gamma(1+\sigma^2/m_2)}{m_2(\kappa_s\cos^4\theta + \kappa_{s_I}\sin^4\theta + \frac{3}{2}\sin^2 2\theta + 3\sigma^2/m_2^2)}.$$

By letting $\gamma = \kappa_{s_R}m_2 = m_{R4}/m_2$ (which implies that the real part is the reference direction of the constellation derotation), this becomes

$$r^2 = \begin{cases} \dfrac{1+\sigma^2/m_2}{1+\dfrac{3\sigma^2}{\kappa_s m_2^2}} & \text{for } \theta = 0,\ \pi \\[2em] \dfrac{1+\sigma^2/m_2}{\dfrac{\kappa_{s_I}}{\kappa_{s_R}}+\dfrac{3\sigma^2}{\kappa_s m_2^2}} & \text{for } \theta = \pi/2,\ 3\pi/2 \end{cases}.$$

Notice that when $\sigma^2 = 0$,

$$r^2 = \begin{cases} 1 & \text{for } \theta = 0,\ \pi \\ \frac{\kappa_{s_R}}{\kappa_{s_I}} & \text{for } \theta = \pi/2,\ 3\pi/2 \end{cases}$$

This indicates that by monitoring $||\mathrm{f}||^2$, the possibly undesirable convergence to the local minima at $\theta = \pi/2$ and at $\theta = 3\pi/2$ can be detected.

3. EXAMPLES

3.1. For QAM sources

For an ordinary QAM source, the real and the imaginary components are independent, but with identical distributions. Thus the received signal satisfies the assumptions after proper gain control, that is, when the gain of the real channel and imaginary channel are equal. The cost function in (2) can be simplified to

$$J = \frac{m_2^2}{2}(3-\kappa_s)\sin^2 2\theta + m_4 + 3\sigma^2 - 2\gamma(m_2+\sigma^2) + \gamma^2 \tag{6}$$

which is plotted in Figure 2 as a function of the kurtosis of the source. Notice that the cost surface is symmetric about $\pi/2$ rotation due to the statistical homogeneity of the real and the imaginary components of the QAM signal.

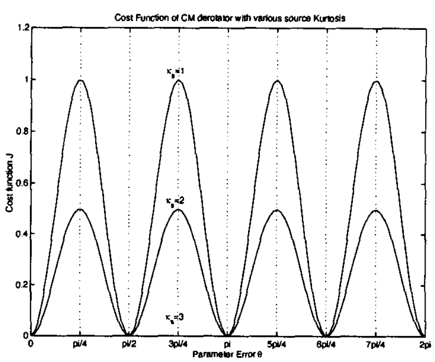


Figure 2: The Cost function of the CMD for QAM signals as a function of source kurtosis

3.2. For signals with $\kappa_{s_I} \neq \kappa_{s_Q}$

Let assume a souce of which kurtosis of the imaginary component is greater than that of its real component,i.e. $\kappa_{s_I} > \kappa_{s_R}$ In this case the asymmetry of source statistics induces an asymmetry in the cost function as shown in Figure 3.

4. CMD IN THE PRESENCE OF NON-IDEALITIES

A received signal suffering from ISI, or one generated from a source with an inherent In-phase/Quadrature dependency may violate assumptions ii) and iii). However, for most communication source singals, we can assume $E\left\{s_R^3 s_I\right\} = E\left\{s_R s_I^3\right\} = E\left\{s_R s_I\right\}$=0 even in the presence of ISI or when the in-phase components are

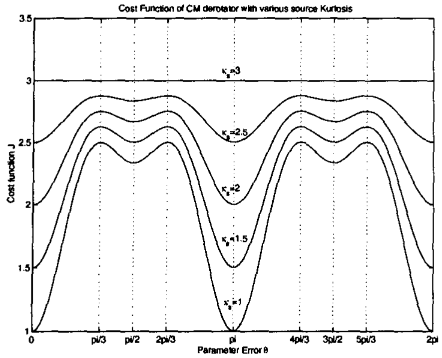


Figure 3: The Cost function of CM derotator for $\kappa_{s_I} > \kappa_{s_R}$.

dependent on the quadrature components. With these minimal assumptions on the source statistics, the cost function of CMD expands as

$$\begin{aligned} J(\theta) &= m_{R4}\cos^4\theta + m_{I4}\sin^4\theta + \frac{3}{2}E\left\{s_R^2 s_I^2\right\}\sin^2 2\theta \\ &\quad -2\gamma(E\left\{s_R^2\right\}\cos^2\theta + E\left\{s_I^2\right\}\sin^2\theta) + c. \qquad (7) \end{aligned}$$

where c is a constant independent of θ. For most communication source signals, The following two subsections focus on two important cases where the cost function suffers from ISI and from such dependencies.

4.1. Inter-Symbol Interference

In the presence of ISI, the most significant distortion is due to $E\left\{s_R^2\right\} \neq E\left\{s_I^2\right\}$, since the power loss of the real channel and of the imaginary channel may be different. In place of assumption iii), suppose that

$$m_{R2} = E\left\{s_R^2\right\} = \rho E\left\{s_I^2\right\} = m_{I2} \qquad (8)$$

due to the ISI. Then the cost function (4) in the presence of ISI can be written as

$$\begin{aligned} J_{ISI} &= m_{R4}\cos^4\theta + m_{I4}\sin^4\theta + \frac{3}{2}\rho m_{R2}^2\sin^2 2\theta \\ &\quad -2\gamma m_{R2}(\rho-1)\sin^2\theta + c. \qquad (9) \end{aligned}$$

As before, set $\gamma = 0$. Then

$$\frac{\partial J}{\partial\theta} = 2m_{R2}^2\sin 2\theta\left\{(\kappa_{s_R} + \frac{\kappa_{s_I}}{\rho^2} - 6\rho)\sin^2\theta + 3\rho - \kappa_{s_R}\right\} \quad (10)$$

Recall that κ_{s_R} and κ_{s_I} are the kurtosis of the received signal, thus are often larger than those of the original QAM source. As $\rho \to 0$, the sign of $\partial J_{ISI}/\partial\theta$ in (10) changes. This swaps the local minima and local maxima, and thus can result in a catastrophic performance degradation. Figure 4 shows the cost surface of a non-QAM signal for $\rho = 1, 0.85$, and 0.7. Notice that when $\rho = 0.7$ local minima appear at undesirable locations. It is interesting that the performance degradation of the CMD does not depend on the severity of the ISI, but on the power mismatch between the real channel and imaginary channels.

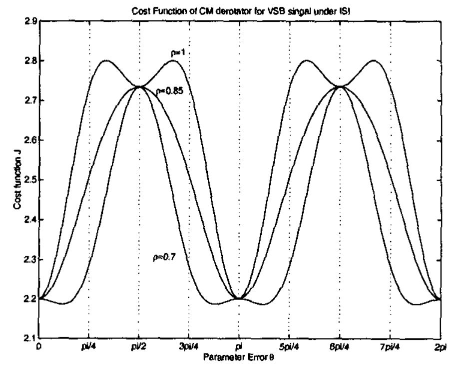


Figure 4: The Performance of CMD for $\kappa_{s_R} > \kappa_{s_I}$ ISI channel

4.2. In-phase and Quadrature Dependency

Assume that the source has identical in-phase and quadrature components but an inherent dependency between them (such as in a modified QAM). Thus, assumption ii)(a) is replaced by

$$E\left\{s_R^2 s_I^2\right\} < E\left\{s_R^2\right\}E\left\{s_I^2\right\} = m_2^2. \qquad (11)$$

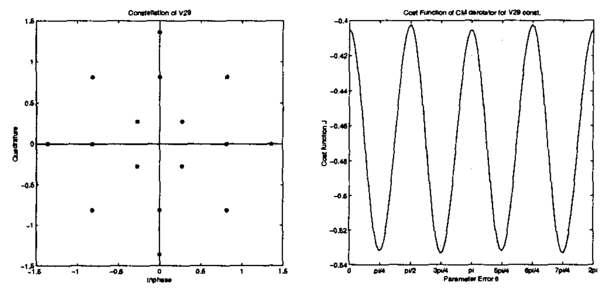
In this scenario, the cost function is

$$J_{dep} = \frac{m_2^2}{2}\left(\frac{3E\left\{s_R^2 s_I^2\right\}}{m_2^2} - \kappa_s\right)\sin^2 2\theta + c. \qquad (12)$$

When $\kappa_s > 3E\left\{s_R^2 s_I^2\right\}/m_2^2$, the sign of the $\sin^2 2\theta$ term in (12) changes to negative. Thus the cost function has local minima at $\theta = \pi/4$ and at $\theta = \pi/3$. For example, the V29 constellation for a telephone modem line (Figure 5-a) satisfies this condition, since for V29 sources

$$\begin{aligned} m_2 = 1/2, \quad m_4 &= \tfrac{5\cdot 29}{3^5} \approx 0.5967, \\ E\left\{s_R^2 s_I^2\right\} &= \tfrac{2\cdot 41}{3^6} \approx 0.1125, \\ \kappa_s \approx 2.3868 \;>\; 3\frac{E\left\{s_R^2 s_I^2\right\}}{m_2^2} &\approx 1.3498 \end{aligned}$$

Thus, when using a constellation such as V29, the value of ϕ which minimizes the cost $J(\phi)$ of (12) is offset from the projection onto the real axis by 45 degrees, that is, $\phi = \Phi + \frac{\pi}{4}$. Hence when applying the algorithm to such a constellation, this offset can be accounted for a priori. Alternatively, it is possible to change the sign on the stepsize of the algorithm, effectively searching for the direction which maximizes the dispersion rather than minimizing it. This simply inverts the cost function, turning the peaks of Figure (Figure 5-a) into valleys, and the valleys into peaks. In this situation, ϕ again converges to Φ.



a) V29 Constellation   b) Cost function

Figure 5: The CMD cost for V29 signal

5. TRACKING ABILITY OF THE CMD

In many applications, the phase offset may drift over time due to the frequency offset of the carrier loop. This subsection investigates the tracking ability of the CMD in the presence of a linear phase offset.

Assume the phase offset is drifting linearly at a rate Ω, i.e. the true phase offset at the k-th update is given by $\Phi + k\Omega$. Define

$$\theta_k := \Phi + k\Omega - \phi_k, \tag{13}$$

the deviation of the estimated parameter of the CMD ϕ_k from the true phase offset. From the update equation, (w.l.og. $\gamma = 0$)

$$\begin{aligned} \theta_{k+1} &= \theta_k + \Omega - \mu\Re(ye^{-i\phi_k})^2\Re(ye^{-i\phi_k})\Im(ye^{-i\phi_k}) \\ &= \theta_k + \Omega - \mu\Re(se^{i\theta_k})^2\Re(se^{i\theta_k})\Im(se^{i\theta_k}). \end{aligned} \tag{14}$$

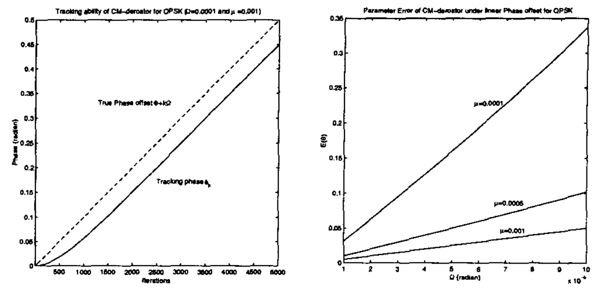
By taking the ensemble average of the above non-linear dynamic system

$$\begin{aligned} E(\theta_{k+1}) &= E(\theta_k) + \Omega + \mu E\left\{\Re(se^{i\theta_k})^2\Re(se^{i\theta_k})\Im(se^{i\theta_k})\right\} \\ &= E(\theta_k) + \Omega - \mu\frac{m_2^2}{2}E\{\sin 2\theta_k\,((\kappa_{s_R} + \kappa_{s_I} \\ &\quad -6)\sin^2\theta_k + 3 - \kappa_{s_R})\}. \end{aligned} \tag{15}$$

Assume a steady state of the above system as shown in Figure 6-a) so that $E(\theta_{k+1}) = E(\theta_k)$, and furthermore assume that Ω is small enough to validate the first order approximation in the expectation term in (15). Then

$$E(\theta_k) \approx \frac{\Omega}{\mu m_2^2(3 - \kappa_{s_R})}. \tag{16}$$

This agrees well with simulation results for small Ω (for example Figure 6-b).



a) Trace   b) $E(\theta_k)$ as $k \to \infty$

Figure 6: Tracking ability of CMD for QPSK

6. CONCLUSION

The ability to simply and accurately accomplish phase rotation may be useful in a variety of situations. We have proposed a simple phase derotation algorithm inspired by the ideas of the constant modulus property restoral algorithm. We have confirmed that the proposed CMD works as expected in ideal situations, and studied its behavior in less ideal situations.

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