Characterization of Multipath Distortion of FSK Signals

Wonzoo Chung, C. R. Johnson, Jr., and M. J. Ready

***Abstract*—In this letter, we characterize the nonlinear distortion of frequency-shift keying (FSK) signals due to a multipath channel as a linear time varying channel. The dynamic range of time variation of this time-varying channel model is small for a mild multipath channel and large for a severe multipath channel. This characterization justifies the use of a decision directed adaptive linear equalizer for FSK signals after frequency demodulation in order to mitigate the distortion due to a multipath channel.**

***Index Terms*—Blind equalization, FSK modulation, multipath distortion.**

I. Introduction

MODERN digital communication systems utilize various digital signal processing (DSP) techniques in order to enhance receiver performance. Digital filtering for channel equalization is perhaps the most successful example, where an adaptive filter mitigates inter symbol interference of linearly modulated signals such as quadrature amplitude modulation (QAM). For linearly modulated signals, the distortion due to a multipath channel is well modeled by a time invariant linear filter. Hence, equalization is basically achieved by deconvolution of the baud spaced or fractionally spaced sampled received signal [6]. However, for nonlinear modulation schemes such as frequency-shift keying (FSK) the multipath distortion cannot be modeled as a time invariant linear filter, and a simple linear equalization scheme for this distortion is considered infeasible.

In this letter, we show that adaptive linear filtering can be used still as an effective means of mitigating the multipath distortion of FSK signals by modeling this nonlinear distortion as a time varying linear channel in the absence of channel noise. The coefficients of this time varying channel model are influenced by the magnitude of the multipath channel coefficients (time invariant quantity) and phase terms of the multipath channel and FSK signals (time variant quantity). Thus, the time span of this time-varying channel model is the same as the time span of the multipath channel for FSK signals, and the dynamic range of the time variation depends on the severity of the multipath channel. Decision-directed LMS (DD-LMS) can be applied in order to equalize the short-term time average channel of a modestly time varying channel model. For a more severe multipath channel additional blind adaptive equalization, such as via constant modulus algorithm (CMA) [7], can be applied directly on the predemodulated FSK signal in order to reduce the severity of the multipath channel, as done experimentally in [5].

Manuscript received November 10, 2000; revised October 9, 2001. This work was supported in part by NSF Grant ECS-9811297 and Applied Signal Technology. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. G. Ramponi.

W. Chung and C. R. Johnson, Jr. are with the School of Electrical and Computer Engineering, Cornell University, Ithaca, NY 14853 USA (e-mail: wonzoo@ee.cornell.edu).

M. J. Ready is with Transcendent Technology, Inc., Sunnyvale, CA 94086.

Publisher Item Identifier S 1070-9908(02)02416-1.

II. Multipath Distortion in FSK System

A. Review of a FSK System

We consider a continuous phase FSK (CPFSK) scheme [4]. The discrete PAM sequence $\{s_n\}$ is converted to a continuous time signal $v(t)$ by a pulse shaping filter $g(t)$[1] with baud interval T

$$v(t) = \sum s_n g(t - nT). \tag{1}$$

Then $v(t)$ is integrated and frequency modulated

$$\psi(t) := \int_{-\infty}^{t} v(\tau)\, d\tau \tag{2}$$

$$u(t) := \exp[j\beta\psi(t) + j\zeta_0] \tag{3}$$

where β is a constant called the modulation index. Without loss of generality, we assume $\beta = 1$ and the phase offset $\zeta_0 = 0$ henceforth. Assuming a linear channel $\mathbf{c}$ and absence of channel noise, the received signal is represented as

$$r(t) = \mathbf{c}(t) \star \mathbf{u}(t) \tag{4}$$

where $\star$ denotes convolution. Suppose that the multipath delays of $\mathbf{c}(t)$ are integer multiples of a constant t_0, i.e.,

$$\mathbf{c}(t) = \sum_{i=0}^{i=N_c} \rho_i e^{j\phi_i} \delta(t - it_0) \tag{5}$$

where ρ_i and ϕ_i denote the magnitude and phase, respectively.

In the receiver, the received signal $r(t)$ is demodulated by extracting an estimate of $\psi(t)$ as the phase of $r(t)$ and differentiating

$$\hat{\psi}(t) = \angle r(t) \tag{6}$$

$$\hat{v}(t) = \frac{d}{dt}\hat{\psi}(t). \tag{7}$$

Finally, the output of the receiver is given by

$$\hat{s}_n = \hat{v}(t)|_{t=nT}. \tag{8}$$

B. Time-Varying Filtering Model of Multipath Distortion

Now we establish a mathematical model $\mathbf{h}(t)$ for the effect of the multipath channel $\mathbf{c}(t)$ on the relation between the pulse

[1] We assume g include matched filter, i.e., $g = g_T(t) \star g_T^*(-t)$.

shaped source signal $v(t)$ in (1) and the demodulated received signal $\hat{v}(t)$ in (7). From

$$r(t) = \sum_{i=0}^{N_c} \rho_i e^{j(\psi(t-it_0)+\phi_i)} \tag{9}$$

define

$$r_X(t) := \sum \rho_i \cos(\psi(t-it_0)+\phi_i) \tag{10}$$

$$r_Y(t) := \sum \rho_i \sin(\psi(t-it_0)+\phi_i) \tag{11}$$

$$\hat{v}(t) = \frac{d}{dt} \tan^{-1}\left(\frac{r_Y(t)}{r_X(t)}\right) \tag{12}$$

$$= \frac{1}{|r(t)|^2}\left(r_X(t)\dot{r}_Y(t) - r_Y(t)\dot{r}_X(t)\right) \tag{13}$$

where $\dot{r}_X(t)$ and $\dot{r}_X(t)$ are time derivatives of r_X and r_Y, respectively, i.e.

$$\dot{r}_X(t) = -\sum \rho_i \sin(\psi(t-it_0)+\phi_i)v(t-it_0) \tag{14}$$

$$\dot{r}_Y(t) = \sum \rho_i \cos(\psi(t-it_0)+\phi_i)v(t-it_0). \tag{15}$$

Define a discrete time varying linear filter $\mathbf{h}(t)$ via its components

$$\mathbf{h}(t)_i := \frac{\rho_i}{|r(t)|^2}\left[r_X(t)\cos(\psi(t-it_0)+\phi_i) + r_Y(t)\sin(\psi(t-it_0)+\phi_i)\right]. \tag{16}$$

Then, $\hat{v}(t)$ can be expressed as

$$\hat{v}(t) = \mathbf{h}^T(t)\cdot\mathbf{v}(t) \tag{17}$$

where $\mathbf{v}(t)$ is a vector drawn from $v(t)$ defined as

$$\mathbf{v}(t) = [v(t)\cdots v(t-it_0)\cdots v(t-iN_c)]. \tag{18}$$

When $\hat{v}(t)$ is sampled at $T_s = t_0$, (17) reveals a linear time-varying channel model of the FSK distortion between $\{\hat{v}(t)|_{t=kt_0}\}$ and $\{v(t)|_{t=kt_0}\}$ via $\mathbf{h}(t)$.

C. *Properties of Time-Varying Channel Model*

Here, we examine two properties of $\mathbf{h}(t)$ in order to characterize the distortion of FSK signals due to a multipath channel.

Property 1:

i) $\sum \mathbf{h}(t)_i = 1$ as long as $|r(t)| \neq 0$. Thus, $\mathbf{h}(t) \neq 0$, unless $\mathbf{c}(t) = 0$.

ii) Denote

$$\psi_{ik}(t) := \psi(t-it_0) - \psi(t-kt_0) + \phi_i - \phi_k \tag{19}$$

then $\mathbf{h}(t)$ can be compactly described by $\{\rho_i\}$, the magnitude coefficients of the channel, and the phase terms $\{\cos(\varphi_{ik}(t))\}_{i,k=1,\ldots,N_c}$, i.e.,

$$\mathbf{h}(t)_i = \rho_i \frac{\sum_k \rho_k \cos(\varphi_{ik}(t))}{\sum_i\sum_k \rho_i\rho_k\cos(\varphi_{ik}(t))}. \tag{20}$$

Proof: The proof of 1) follows from using (10) and (11) in (16) to reveal

$$\sum_i \mathbf{h}(t)_i = \frac{1}{|r(t)|^2}\left[r_X(t)r_X(t) + r_Y(t)r_Y(t)\right] = 1 \tag{21}$$

as long as $|r(t)| \neq 0$. Since $\sum h_i(t) = 0$ is a necessary condition for $\mathbf{h}(t) = 0$, $\mathbf{h}(t) \neq 0$ unless $\mathbf{c}(t) = 0$. The proof of ii) follows from trigonometric identities. ■

Since $\mathbf{h}(t)$ is a function of $\{\cos(\varphi_{ik}(t))\}$, the time variation of $\mathbf{h}(t)$ is determined by the time variation of $\{\cos(\varphi_{ik}(t))\}$ and their contribution to $\mathbf{h}(t)$ through $\{\rho_i\rho_k\}$. On the other hand, we have

$$\varphi_{ik}(t) = \int_{\tau=t-it_0}^{\tau=t-kt_0} v(\tau)\,d\tau + \phi_i - \phi_k. \tag{22}$$

Since $v(\tau)$ is generated by an i.i.d. source sequence $\{s_n\}$, $\cos(\varphi_{ik}(t))$ is also observed to behave like a random source independent from t_0 when $t_0 > T$ (recall that T is the baud interval). Therefore, $\mathbf{h}(t)$ can be viewed as a filter randomly changing over its mean value $E\{\mathbf{h}(t)\}$. Although $E\{\mathbf{h}(t)\}$ is mainly determined by $\{\rho_i\}$, it is difficult to calculate explicitly. Thus, we focus on the range of $\mathbf{h}(t)$ in terms of severity of the channel.

Property 2: Let us consider channels satisfying a condition that the amplitude of one tap, denote its index as i_c, dominates that of all other taps. Explicitly, the products of any two other taps $\{\rho_l\rho_m\}_{i_c\neq l,\,m}$ are considered ignorable in comparison with the main tap ρ_{i_c}. For such channels the dynamic range of $\mathbf{h}(t)$ is large for a severe multipath channel and small for a mild multipath channel.

Proof: Applying the following bound

$$\sum_k \rho_k^2 - \sum_{k\neq l}\rho_k\rho_l \leq \sum_i\sum_k \rho_i\rho_k\cos(\varphi_{ik}(t)) \leq \sum_k \rho_k^2 + \sum_{k\neq l}\rho_k\rho_l \tag{23}$$

and the following approximation justified by the assumption that $\{\rho_l\rho_m\}_{i_c\neq l,\,m}$ are ignorable in comparison with the main tap ρ_{i_c}

$$\sum_k \rho_k^2 - \sum_{k\neq l}\rho_k\rho_l \approx \left(\rho_{i_c} - \sum_{k\neq i_c}\rho_k\right)^2 \tag{24}$$

to (20) yields

$$\frac{\rho_{i_c}^2 - \rho_{i_c}\sum_{k\neq i_c}\rho_k}{\left(\rho_{i_c} + \sum_{k\neq i_c}\rho_k\right)^2} \leq \mathbf{h}(t)_{i_c} \leq \frac{\rho_{i_c}^2 + \rho_{i_c}\sum_{k\neq i_c}\rho_k}{\left(\rho_{i_c} - \sum_{k\neq i_c}\rho_k\right)^2}. \tag{25}$$

From (25), we can observe that as a channel becomes severe $(\rho_{i_c} - \sum_{k\neq i_c}\rho_k \to 0)$, the upperbound of $\mathbf{h}(t)_i$ dramatically increases. ■

As illustrated in property 2, our use of the label of multipath severity implies that the magnitudes of secondary path gains approach that of the largest.

D. *Example: Two Tap Channel Case*

Consider a two-tap multipath channel $\mathbf{c}$ described by

$$\mathbf{c}(t) = \delta(t) + \rho e^{j\phi}\delta(t-t_0). \tag{26}$$

From (3) and (4), as in (9),

$$|r(t)|^2 = 1 + 2\rho\cos(\varphi(t)) + \rho^2 \tag{27}$$

and (16) and (20)

$$\mathbf{h}(t) = \left[\frac{1}{1+\rho\cos(\varphi(t))}\;\frac{\rho\cos(\varphi(t))}{1+\rho\cos(\varphi(t))}\right]. \tag{28}$$

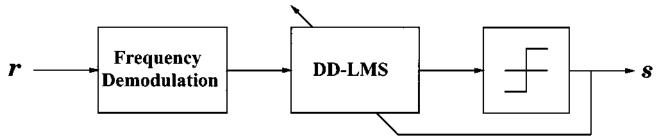

Fig. 1. Post-D filter.

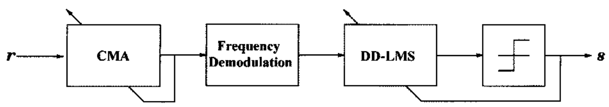

Fig. 2. Pre-D filter and post-D filter.

Notice that a destructive interference occurs when the two taps have the same magnitude with opposite phase ($\rho = 1$ and $\varphi = \pi$), such that $|\mathbf{h}(t)_0| \rightarrow \infty$ and $|\mathbf{h}(t)_1| \rightarrow \infty$. This simple two tap channel model yields a tighter bound on $\mathbf{h}(t)_0$ than (25)

$$\frac{1}{1+\rho} \leq \mathbf{h}(t)_0 \leq \frac{1}{1-\rho}. \tag{29}$$

III. Equalization

Now we consider equalization of FSK signals based on the channel modeling developed in the preceding section.

A. Blind Adaptive Post-D and Pre-D Filtering

Figs. 1 and 2 illustrate blind adaptive equalization schemes for CPFSK presented in [5]. Fig. 1 shows an equalizer using DD-LMS, which is referred to as a post-D filter. Notice that as a result of the time-varying channel model (17) the length of the post-D filter permitting perfect equalization is the same as that of a conventional $T/2$ linear equalizer for a linear channel [2]. This DD-LMS equalizer tends to converge to equalize $E\{\mathbf{h}(t)\}$ and provide significant performance gain as shown in Fig. 3, discussed below.

For severe multipath channels, in order to deal with wide range of $\mathbf{h}(t)$, CMA equalization can be applied to the received FM signal before demodulation (which is referred to as a pre-D filtering). However, CMA requires an i.i.d. source for successful equalization [2], while the FSK signal is not an i.i.d. source in general (since it is generated through an integration operator). Although the pre-D CMA filter may not achieve perfect equalization for the FSK signal, it can mitigate enough multipath distortion so that post-D DD-LMS algorithm can equalize remaining $\mathbf{h}(t)$.

B. Simulation Results

In conclusion, we examine the performance of DD-LMS. Channels A and B are two tap channels with the delay $t_0 = 2T_s$ ($T_s = T/2$, where T is baud interval) and $\rho = 0.2$ and $\rho = 0.5$ respectively. We used raised cosine pulse shaping for 2-level FSK. Fig. 3 illustrates the performance gain of the $T/2$ post-D DD-LMS filter of length 10 in the presence of noise (SNR 4–20 dB) with the step size $\mu = 0.001$. For more realistic simulation results refer to [1], [5].

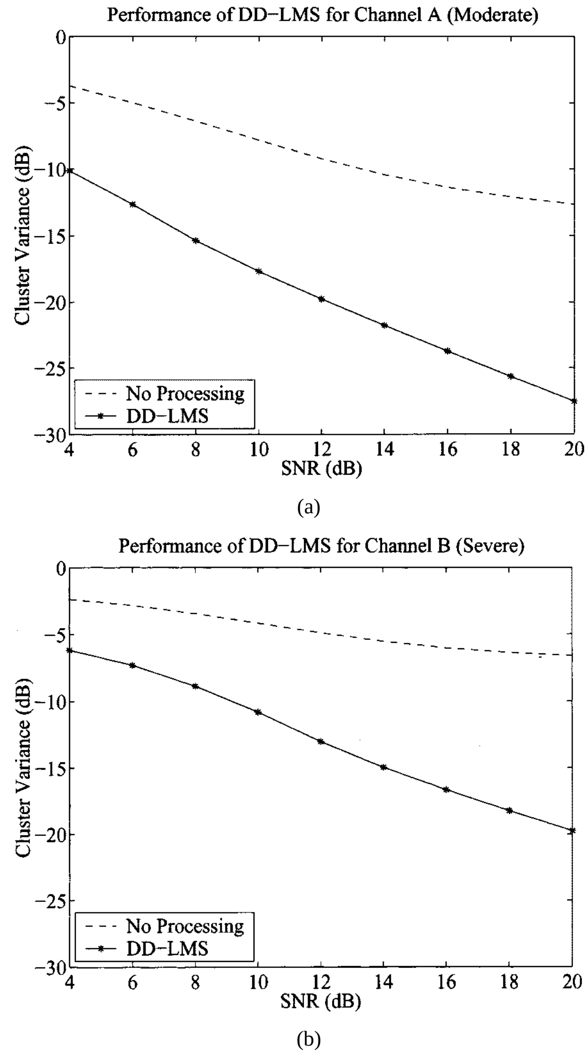

Fig. 3. (a) Performance of Post-D DD-LMS for a moderate channel c= [1 0.2]. (b) Performance of post-D DD-LMS for a severe channel c= [1 0.5].

References

[1] W. Chung, R. K. Martin, M. Ready, and C. R. Johnson, Jr., "Blind adaptive equalizer for FSK systems based on a characterization of multipath on the FSK signals," in *Proc. 2000 CISS*, Princeton, NJ, Mar. 2000, pp. 28–32.

[2] C. R. Johnson, Jr., P. B. Schniter, T. S. Endres, J. M. Behm, D. R. Brown, and R. A. Casas, "Blind equalization using the constant modulus criterion: A review," *Proc. IEEE*, vol. 86, pp. 1927–1950, Oct. 1998.

[3] P. F. Panter, *Modulation, Noise, and Spectral Analysis*. New York: McGraw-Hill, 1965.

[4] J. G. Proakis and M. Salehi, *Communication Systems Engineering*. Englewood Cliffs, NJ: Prentice-Hall, 1994.

[5] M. J. Ready and J. Harp, "Performance improvements achieved by equalizing intermediate rate FSK signals," in *Proc. Asilomar Conf. Circuits, Systems, and Computers*, Pacific Grove, CA, Nov. 1998, pp. 379–383.

[6] L. Tong, G. Xu, and T. Kailath, "Blind identification and equalization based on second-order statistics: A time domain approach," *IEEE Trans. Inform. Theory*, vol. 40, pp. 340–349, Mar. 1994.

[7] J. R. Treichler and B. G. Agee, "A new approach to multipath correction of constant modulus signals," *IEEE Trans. Acoust, Speech, Signal Processing*, vol. ASSP-31, pp. 459–472, Apr. 1983.