Blind Adaptive Cross-Pole Interference Cancellation Using Fractionally-Spaced CMA

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Abstract

Cross-pole interference (CPI) cancellation is a specific type of source separation problem in a high-rate digital communication system which utilizes dual polarized carriers in order to achieve frequency reuse. In this paper, we analyze a fractionally-spaced (FS) blind adaptive cross-pole interference (CPI) cancellation scheme using the constant modulus algorithm (CMA). First, we study zero forcing solutions and minimum mean-squared error solutions of FS CPI equalizer on a cross coupled channel with a certain diversity. Second, we show close analogies between the FS-CMA CPI equalizer and the conventional FS-CMA blind adaptive equalizer. Thus existing results on FS-CMA can be applied to describe the important behavior of the FS-CMA CPI equalizer, for example, the agreement between CM-solutions and zero-forcing (ZF) solutions in the absence of channel noise and perfect modelling and the robustness of the FS-CMA CPI equalizer in more realistic situations (wchich include channel noise and undermodelling). Finally, we address the initialization of this blind algorithm by using a novel expression of the FS-CMA CPI cost function.

1 Introduction

In modern communication systems, the demand for transmitting extensive information through channels of limited bandwidth has encouraged the search for an efficient transmission scheme exploiting both spectral and spatial resources. Especially in digital microwave radio systems, such an efficient system can be realized by frequency reuse of two orthogonally polarized carrier waves at the same frequency. Two mutally independent source sequences $\mathbf{s}_1$ and $\mathbf{s}_2$ are transmitted simultaneously on vertical and horizontal polarizations at the same nominal carrier frequency and received by two appropriately polarized antennas. Armed with this technique, theoretically, one can transmit twice the amount of information through a fixed bandwidth channel. However, it has been observed that in practice the performance of the system tends to be degraded due to the interference (including cross-pole) of signals passing through physical channels.

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Interference is caused by not only the bandwidth of the channel (inter symbol interference (ISI)) but also by the structure of orthogonal polarization: refraction and reflection of carrier waves during propagation allow only mixed signals to reach the receiver antennas. The structural interference is referred to as *cross-polarization interference* (CPI) and modelled as a linear addition of two source sequences. Recovering signals despite CPI, as well as ISI, is essentially a source separation problem. One may apply conventional dual finite impulse response (FIR) filter equalization techniques based on estimation of second order statistics estimation or training sequence adaptation [7, 8].

A blind adaptive CPI cancellation scheme based on high order source statistics has been proposed by Treichler et al. and implemented recently [10, 11, 12]. This scheme is a natural extension of the constant modulus algorithm (CMA) to a dual FIR filter equalizer. It also can be applied to other blind signal separation problems in digital communication systems, such as sidelobe canceling. Although in experiments CMA CPI shows satisfactory robust performance [12], one issue yet to be resolved is the initialization of this blind algorithm. In this paper, we consider CPI cancellation with a fractionally spaced equalizer and present an analysis of fractionally spaced CMA CPI focusing on the effect of initialization.

In Section 2, we establish a fractionally spaced CPI cancellation model. With a generalized version of a conventional perfect equalization assumption on a fractionally spaced receiver, FS CPI with an FIR channel model admits FIR equalizer solutions. We study zero-forcing (ZF) solutions and minimum mean squared error (MMSE) solutions in Section 3. FS CPI equalization using CMA is analyzed by analogy to conventional FS CMA equalization and the initialization issue is addressed in Section 4.

2 Fractionally Spaced CPI Cancellation

2.1 System model

Figure 1 models cross-polarization interference in a communication system using vertically and horizontally polarized antennas. Two independent source sequences $\mathbf{s}_1$ and $\mathbf{s}_2$ are transmitted on horizontally and vertically polarized carrier waves, respectively. FIR filters $\mathbf{c}_{11}$ and $\mathbf{c}_{22}$ represent the direct path between horizontal and vertical polarization antennas, respectively, while $\mathbf{c}_{12}$ and $\mathbf{c}_{21}$ represent cross-coupling terms.

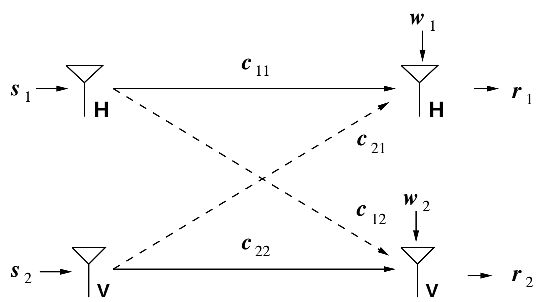


Figure 1. Cross Polarization Interference

The received signal of a fractionally spaced receiver is sampled at a fraction of the baud rate. This way, additional channel information from the fractional sampling can be utilized for equalization. We consider a $T/2$-spaced receiver (i.e., the sampling interval is half of the baud interval T). A $T/2$-spaced CPI equalizer intended to extract one of the transmitted signals is illustrated in Figure 2.

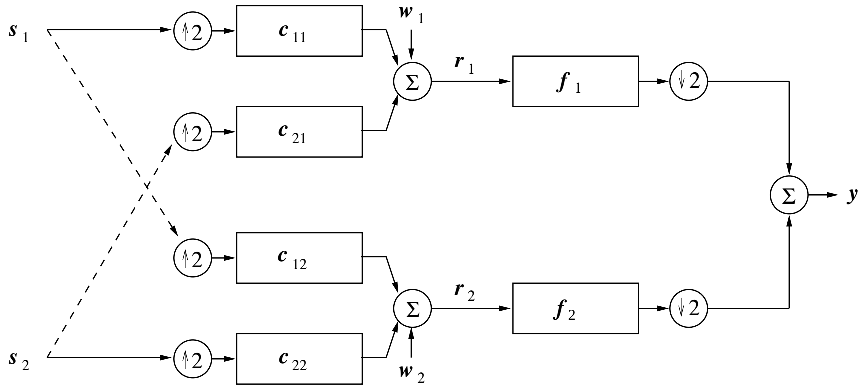


Figure 2. FS CPI equalizer for a desired signal.

For now suppose there is no channel noise $\mathbf{w}_1$ or $\mathbf{w}_2$. Then, for channel paths $\mathbf{c}_{ij}$ and equalizers $\mathbf{f}_k$, the combined baud spaced response can be written as a linear combination of channel convolution matrices C_{ij} and equalizer vectors $\mathbf{f}_i$, i.e., $C_{ij}\mathbf{f}_k$ [5]. Thus in the absence of noise, the output y of the model in Figure 2 is given by

$$y = \mathbf{s}_1^T(C_{11}\mathbf{f}_1 + C_{12}\mathbf{f}_2) + \mathbf{s}_2^T(C_{21}\mathbf{f}_1 + C_{22}\mathbf{f}_2) \tag{1}$$

where C_{11}, C_{12}, C_{21}, and C_{22} are the (row-decimated, baud spaced) convolutional matrices of $\mathbf{c}_{11}, \mathbf{c}_{12}, \mathbf{c}_{21}$ and $\mathbf{c}_{22}$ respectively. We denote the system response vectors of each channel-equalizer combination as

$$\begin{array}{ll} \mathbf{h}_1 := C_{11}\mathbf{f}_1, & \mathbf{h}_2 := C_{21}\mathbf{f}_1 \\ \mathbf{g}_1 := C_{12}\mathbf{f}_2, & \mathbf{g}_2 := C_{22}\mathbf{f}_2 \end{array} \tag{2}$$

We define $\mathbf{q}_1 := \mathbf{h}_1 + \mathbf{g}_1$ and $\mathbf{q}_2 := \mathbf{h}_2 + \mathbf{g}_2$ as the system responses of the FS CPI equalizer for sources $\mathbf{s}_1$ and $\mathbf{s}_2$, and we have the following linear relation:

$$\begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \end{bmatrix} := \begin{bmatrix} \mathbf{h}_1 + \mathbf{g}_1 \\ \mathbf{h}_2 + \mathbf{g}_2 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \end{bmatrix} \tag{3}$$

Define

$$C := \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \tag{4}$$

as an analogy of the (baud spaced) channel convolution matrix for an FS equalizer. C is invertible when the *Schur complement of* C_{11} in C, i.e, $C_{22} - C_{21}C_{11}^{-1}C_{12}$, is non singular [6].

2.2 Assumptions

With a fractionally-spaced equalizer, it has been shown that perfect equalization is possible [9]. We assume the following [5] for the channels $\{\mathbf{c}_{ij}\}$ and the equalizers $\{\mathbf{f}_i\}$ so that C can be invertible (perfect equalization):

A1. The $T/2$-channel models $\{\mathbf{c}_{ij}\}$ are all the same length and satisfy the sub-channel disparity condition by having no roots common to among its sub-channels.

A2. The length of $\{\mathbf{f}_i\}$ satisfies the length condition of perfect equalization for $T/2$-spaced equalization, i.e. the length of equalizer $N_{f_i} = N_c - 2$.

From these assumptions we have invertible channel convolution matrices C_{11}, C_{12}, C_{21}, and C_{22} [5]. Now $\mathbf{h}_2$ and $\mathbf{g}_2$ can be expressed in terms of $\mathbf{h}_1$ and $\mathbf{g}_1$ respectively:

$$\mathbf{h}_2 = A\mathbf{h}_1, \quad \mathbf{g}_2 = B\mathbf{g}_1, \tag{5}$$

where $A := C_{21}C_{11}^{-1}$ and $B := C_{22}C_{12}^{-1}$. The non-singularity of the Schur complement of C_{11}, i.e. $C_{22} - C_{21}C_{11}^{-1}C_{12}$, is equivalent to the non-singularity of $A - B$. We assume that our cross-pole channels have diversity such that

A3. $A - B$ is a non-singular matrix, which guarantees that C is invertible.

3 Second-Order-Statistics Solutions

3.1 Zero-Forcing Solutions

In the absence of noise, the output of the CPI equalizer is given by

$$y = \mathbf{s}_1^T(\mathbf{h}_1 + \mathbf{g}_1) + \mathbf{s}_2^T(\mathbf{h}_2 + \mathbf{g}_2) \tag{6}$$

To equate y to (a delayed version of) $\mathbf{s}_1$, we have two equations

$$\begin{cases} \mathbf{h}_1 + \mathbf{g}_1 = \mathbf{e}_i \\ \mathbf{h}_2 + \mathbf{g}_2 = 0 \end{cases} \text{or equivalently, } \begin{bmatrix} \mathbf{e}_i \\ 0 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \end{bmatrix} \tag{7}$$

Since we assume that C is invertible, (7) yields a unique solution depending on the chosen delay $\mathbf{e}_i := [0 \cdots 0\ 1(i-th)\ 0 \cdots 0]^T$.

$$\begin{cases} \mathbf{f}_1^* = C_{11}^{-1}(B - A)^{-1}B\mathbf{e}_i \\ \mathbf{f}_2^* = -C_{12}^{-1}B^{-1}AC_{11}\mathbf{f}_1^* \end{cases} \tag{8}$$

The derivation of $\mathbf{f}_i^*$ can be found in [6].

3.2 MMSE solution

In this subsection, we now consider the noisy case. Suppose the noises w_1 and w_2 are mutually independent and identically distributed with the same variance σ_w^2, and uncorrelated to the sources s_1 and s_2. The output in the presence of noise is

$$y = \mathbf{s}_1^T\mathbf{q}_1 + \mathbf{s}_2^T\mathbf{q}_2 + \mathbf{w}_1^T\mathbf{f}_1 + \mathbf{w}_2^T\mathbf{f}_2 \tag{9}$$

The minimum mean squared error (MMSE) equalizers $(\mathbf{f}_1^*, \mathbf{f}_2^*)$ are defined as

$$(\mathbf{f}_1^*, \mathbf{f}_2^*) = \arg\min_{\mathbf{f}_1,\mathbf{f}_2} E\left\{\|s_1 - y\|^2\right\} \tag{10}$$

Let $\mathbf{f} := [\mathbf{f}_1^T\ \mathbf{f}_2^T]^T = [f_{1_0}, \cdots, f_{1_{N-1}}, f_{2_0}, \cdots f_{2_{N-1}}]^T$. Since $\mathbf{s}_1$ and $\mathbf{s}_2$ are mutally independent, we treat $\mathbf{s} := [\mathbf{s}_1^T\ \mathbf{s}_2^T]^T$ as a i.i.d. source, and the same for the noise $\mathbf{w} := [\mathbf{w}_1^T\ \mathbf{w}_2^T]^T$. Now we have simplified the expression of y which is similar to that for the FS MMSE equalizer [5].

$$y = \mathbf{s}^T C\mathbf{f} + \mathbf{w}^T\mathbf{f} \tag{11}$$

The MMSE solution $\mathbf{f}^*$ is given by [5]

$$\mathbf{f}^* = (C^H C + \lambda I)^{-1} C^H \mathbf{e}_i \tag{12}$$

where $\lambda = \sigma_{\mathbf{w}}^2/\sigma_{\mathbf{s}}^2$. Or, we can obtain a more explicit expression in terms of C_{ij} by considering:

$$MSE = E\left\{\|s_1 - y\|^2\right\} = \sigma_s^2(\|\mathbf{h}_1 + \mathbf{g}_1 - \mathbf{e}_i\|^2 + \|A\mathbf{h}_1 + B\mathbf{g}_1\|^2) + \sigma_w^2(\|\mathbf{f}_1\| + \|\mathbf{f}_2\|) \tag{13}$$

By setting $\frac{\partial}{\partial h_1} MSE$ and $\frac{\partial}{\partial g_1} MSE$ to zero, we obtain the following equations.

$$\left\{ \begin{array}{l} (I + A^H A + \lambda C_{11}^{-H} C_{11}^{-1})\mathbf{h}_1^* + (I + A^H B)\mathbf{g}_1^* - \mathbf{e}_i = 0 \\ (I + B^H B + \lambda C_{12}^{-H} C_{22}^{-1})\mathbf{g}_1^* + (I + B^H A)\mathbf{h}_1^* - \mathbf{e}_i = 0 \end{array} \right. \tag{14}$$

Because of the non symmetric terms $A^T B$ and $B^T A$, the solutions cannot be expressed in a simple form. From some rather straightforward calculations, we obtain:

$$\begin{aligned} \mathbf{f}_1^* &= C_{11}^{-1} \frac{I - (I + A^H A + \lambda C_{11}^{-H} C_{11})^{-1}}{(I + A^H A + \lambda C_{11}^{-H} C_{11})^{-1} - (I + B^H B + \lambda C_{12}^{-H} C_{12}^{-1})^{-1}(I + A^H B)} \mathbf{e}_i & (15) \\ \mathbf{f}_2^* &= C_{12}^{-1} \frac{A^H A - B^H A + \lambda C_{11}^{-H} C_{11}^{-1}}{B^H B - A^H B + \lambda C_{12}^{-H} C_{12}^{-1}} C_{11} \mathbf{f}_1^* & (16) \end{aligned}$$

where the fraction denotes left inversion (i.e. $\frac{P}{Q} := Q^{-1}P$). Note that the MMSE solutions in (15) and (16) agree with ZF solutions in (8) in the absence of noise (i.e. $\lambda = 0$).

If a communication system allows training signals, the MMSE solutions shown above can be obtained via the LMS algorithm:

$$\begin{aligned} \mathbf{f}_1^{(n+1)} &= \mathbf{f}_1^{(n)} - \mu(y - s_1)\mathbf{r}_1^* & (17) \\ \mathbf{f}_2^{(n+1)} &= \mathbf{f}_2^{(n)} - \mu(y - s_1)\mathbf{r}_2^* & (18) \end{aligned}$$

where μ is a step size and $\mathbf{r}_i^*$ is the complex conjugated regressor vector of equalizer $\mathbf{f}_i$. Notice that the desired signal is determined by the training sequence, i.e. $\mathbf{s}_1$ could be replaced by $\mathbf{s}_2$.

4 FS-CM Blind Cross-Polarized Interference Equalizer

In many applications a blind equalization scheme is preferred, since training signals degrade channel capacity or may not be available. However, blind equalization based on second-order statistics tends to lack robustness in realistic situations [3]. Adaptive blind equalization using the Constant Modulus Algorithm has gained great interest in industry due to its robustness.

The application of CMA to CPI is a natural extension of CMA which inherits the robustness of CMA. To introduce the CMA CPI equalizer, we review CMA in the following subsection.

4.1 Constant Modulus Algorithm

The constant modulus algorithm is a blind adaptive algorithm which exploits the non-gaussian nature of a typical digital communication source [5]. CMA is designed to minimize the following cost function

$$J(\mathbf{f}) = \frac{1}{4} E\left\{(|y(n)|^2 - \gamma)^2\right\} \tag{19}$$

with the stochastic gradient update equation

$$\mathbf{f}^{(n+1)} = \mathbf{f}^{(n)} - \mu \frac{\partial}{\partial \mathbf{f}} J(\mathbf{f})|_{\mathbf{f}=\mathbf{f}^{(n)}} = \mathbf{f}^{(n)} - \mu y(|y(n)|^2 - \gamma)\mathbf{r}^* \tag{20}$$

where μ is a step size and $\mathbf{r}^*$ denotes the complex conjugated regressor of the equalizer $\mathbf{f}$.

In a fractionally-spaced system, the cost function can be expressed in terms of the baud-spaced system response $\mathbf{h}$ (assuming a BPSK source without loss of generality)[1]:

$$4J_H(\mathbf{h}) = (\|\mathbf{h}\|^2 - 1)^2 + \eta_s I_{\mathbf{h}} \tag{21}$$

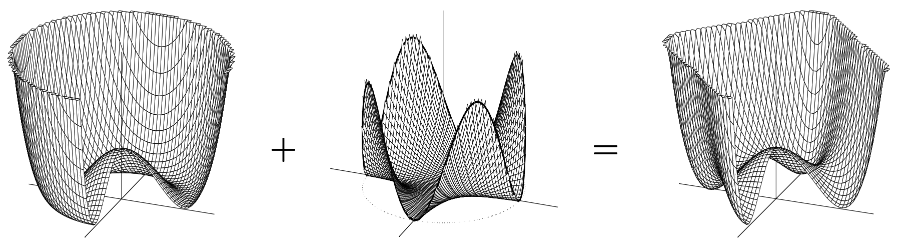
where $I_{\mathbf{h}}$ is a semi-definite real-valued function measuring inter-symbol interference induced by the system response $\mathbf{h}$ defined as

$$I_{\mathbf{h}} := \sum_{i \neq j}^{N\text{-}1} |h_i|^2 |h_j|^2 \tag{22}$$

and η_s characterizes the deviation of the source kurtosis κ_s from that κ_w of a Gaussian sequence.

$$\eta_s := (\kappa_w - \kappa_s) \tag{23}$$

Expression (21) can be understood as a radial and spherical decomposition of the CMA cost function.



a) Radial $(\|\mathbf{h}\|^2 - 1)^2$ b) Spherical $\eta_s I_{\mathbf{h}}$ c) FS-CMA $(\|\mathbf{h}\|^2 - 1)^2 + \eta_s I_{\mathbf{h}}$

Figure 3. Spherical and radial decomposition of CMA cost function

From this representation, some robustness features of CMA can be shown [1, 2, 4]:

- Bad local minima (with large noise gain) tend to disappear as noise increases
- Regions of convergence of good local minima (with small noise gain) expand near the origin, while those of bad local minima shrink.

4.2 CMA CPI equalizer

The structure of the CM CPI equalizer from [12] is shown in Figure 4.

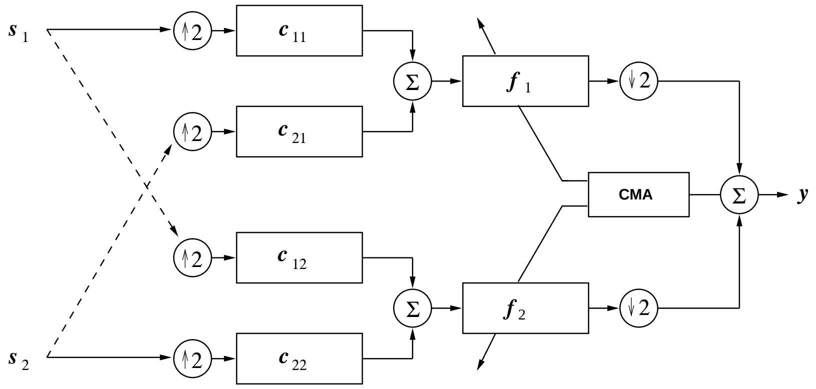


Figure 4. FS CPI cancellation using CMA.

The constant modulus algorithm for CPI uses two stochastic gradient updates: one each for $\mathbf{f}_1$ and $\mathbf{f}_2$.

$$\mathbf{f}_1^{(n+1)} = \mathbf{f}_1^{(n)} - \mu \frac{\partial}{\partial \mathbf{f}_1} J(\mathbf{f}_1, \mathbf{f}_2)|_{\mathbf{f}_1=\mathbf{f}_1^{(n)}, \mathbf{f}_2=\mathbf{f}_2^{(n)}} = \mathbf{f}_1^{(n)} - \mu y(|y(n)|^2 - \gamma)\mathbf{r}_1^* \tag{24}$$

$$\mathbf{f}_2^{(n+1)} = \mathbf{f}_2^{(n)} - \mu \frac{\partial}{\partial \mathbf{f}_2} J(\mathbf{f}_1, \mathbf{f}_2)|_{\mathbf{f}_1=\mathbf{f}_1^{(n)}, \mathbf{f}_2=\mathbf{f}_2^{(n)}} = \mathbf{f}_2^{(n)} - \mu y(|y(n)|^2 - \gamma)\mathbf{r}_2^* \tag{25}$$

where the cost function J is the same as in (19), and $\mathbf{r}_1$ and $\mathbf{r}_2$ denote the regressors of $\mathbf{f}_1$ and $\mathbf{f}_2$ respectively.

The blind nature of CMA raises the following important questions:

- Where are the CM local minima located ?
- How should the equalizer be initialized in order to get a (particular) desired output ?

In the following subsections we answer the above questions.

4.3 CM solutions and ZF solutions

The two update equations (24),(25) for CMA CPI can be expressed by a single equation:

$$\mathbf{f}^{(n+1)} = \mathbf{f}^{(n)} - \mu \frac{\partial}{\partial \mathbf{f}} J(\mathbf{f})|_{\mathbf{f}=\mathbf{f}_1^{(n)}} = \mathbf{f}^{(n)} - \mu y(|y(n)|^2 - \gamma)\mathbf{r}^* \tag{26}$$

where $\mathbf{r} = [\mathbf{r}_1^T \ \mathbf{r}_2^T]^T$ and $\mathbf{f} = [\mathbf{f}_1^T \ \mathbf{f}_2^T]^T$. Given (3), $\mathbf{q}^T := [\mathbf{q}_1^T \ \mathbf{q}_2^T]^T$. Then $\mathbf{q} = C\mathbf{f}$ and we have

$$y = \mathbf{s}^T \mathbf{q} \tag{27}$$

where $\mathbf{s} = [\mathbf{s}_1^T \ \mathbf{s}_2^T]^T$. This expression of the output results in a conventional CMA cost function (assuming a BPSK source) of the form

$$J(\mathbf{q}) = (\|\mathbf{q}\|^2 - 1)^2 + \eta_s I_{\mathbf{q}} \tag{28}$$

with a corresponding update equation. Depending on the initialization of the equalizer $\mathbf{f}_1$ and $\mathbf{f}_2$, we have $\mathbf{q}_1 = \mathbf{e}_i$ and $\mathbf{q}_2 = 0$ or vice versa. In the absence of noise, these CMA solutions match the ZF solutions.

The region of convergence for each delay δ_i is characterized by the matrix $C = [C_{11}C_{12}; C_{21}C_{22}]$ [2]. Since we are interested in the initialization not for each delay but for a desired signal, we consider another expression of the CMA CPI cost function:

FS-CMA CPI Cost Function: The FS-CMA CPI cost function can be expressed in terms of $\mathbf{q}_1$ and $\mathbf{q}_2$:

$$J = \left(\|\mathbf{q}_1\|^2 + \|\mathbf{q}_2\|^2 - 1\right)^2 + 2\eta_s\|\mathbf{q}_1\|^2\|\mathbf{q}_2\|^2 + \eta_s I_{\mathbf{q}_1} + \eta_s I_{\mathbf{q}_2} \tag{29}$$

(29) can be interpreted as follows:

1) Ignoring $\eta_s I_{\mathbf{q}_1} + \eta_s I_{\mathbf{q}_2}$ terms, J is a CM cost function of $\|\mathbf{q}_1\|$ and $\|\mathbf{q}_2\|$. It implies

$$\begin{array}{l} \|\mathbf{q}_1\| = 1 \text{ and } \|\mathbf{q}_2\| = 0 \text{ or} \\ \|\mathbf{q}_1\| = 0 \text{ and } \|\mathbf{q}_2\| = 1 \end{array} \tag{30}$$

2) For $\mathbf{q}_2 = 0$ ($\mathbf{q}_1 = 0$), J is a conventional CMA cost function for $\mathbf{q}_1$ ($\mathbf{q}_2$). Thus we have $\mathbf{q}_1 = \mathbf{e}_i$ and $\mathbf{q}_2 = 0$ or vice versa.

This interpretation indicates that the norm of $\mathbf{q}_1$ and $\mathbf{q}_2$ would be of primary concern in the initialization of CPI CMA, which reduces a multi-dimensional problem into a two dimensional problem.

4.4 Initialization Issue

To describe a closed form expression for the regions of convergence of a given desired signal is almost impossible because of the complicated fourth order nature of the cost function. In this paper we limit our analysis to a special case that matches the experimental setup in [12]. From the observation that CMA CPI can be roughly understood as a CMA on $\|\mathbf{q}_1\|$ and $\|\mathbf{q}_2\|$, we establish regions of convergence for a desired signal in terms of $\|\mathbf{f}_1\|$ and $\|\mathbf{f}_2\|$. To do that, we need a strong assumption on the channel. We consider a class of channels which approximately satisfy the following condition on $\|\mathbf{q}_1\|$ and $\|\mathbf{q}_2\|$.

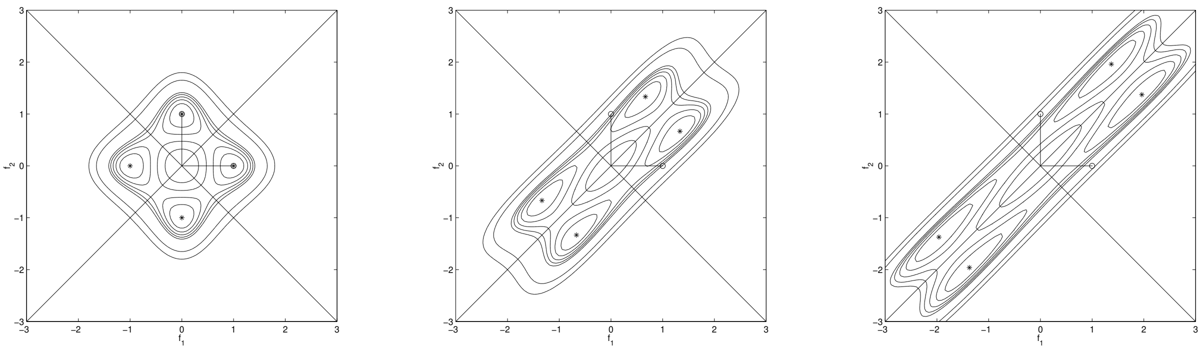
$$\begin{bmatrix} \|\mathbf{q}_1\| \\ \|\mathbf{q}_2\| \end{bmatrix} = \begin{bmatrix} 1 & -\alpha \\ -\alpha & 1 \end{bmatrix} \begin{bmatrix} \|\mathbf{f}_1\| \\ \|\mathbf{f}_2\| \end{bmatrix} \tag{31}$$

where α is labeled as cross power ratio, $0 \leq \alpha \leq 1$, and roughly represents the cross-pole interference power. This class includes simple channels such as scalar channels and a wide band channel of which response can be approximated by a discrete delta function with delays. Let's denote $\bar{C} := [1 \ -\alpha \ ; -\alpha \ 1]$.

The regions of convergence of CMA for $\|\mathbf{q}_1\|$ and $\|\mathbf{q}_2\|$ are equally divided hyper-cones (Figure 5 a) and those for $\|\mathbf{f}_1\|$ and $\|\mathbf{f}_2\|$ are characterized by singular decomposition of $\bar{C}^{-1}$ ("rotation" and "dilation") [2]

$$\begin{bmatrix} 1 & -\alpha \\ -\alpha & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \cos(\frac{\pi}{4}) & -\sin(\frac{\pi}{4}) \\ \sin(\frac{\pi}{4}) & \cos(\frac{\pi}{4}) \end{bmatrix} \begin{bmatrix} \frac{1}{1+\alpha} & 0 \\ 0 & \frac{1}{1-\alpha} \end{bmatrix} \begin{bmatrix} \cos(\frac{\pi}{4}) & \sin(\frac{\pi}{4}) \\ -\sin(\frac{\pi}{4}) & \cos(\frac{\pi}{4}) \end{bmatrix}$$

The cost function is stretched by $\frac{1}{1-\alpha}$ in $x+y$ direction and shrunk by $\frac{1}{1+\alpha}$ in $x-y$ direction, and thus the noise gain of equalizer ($\|f_k\|$) increases. However, the symmetry of $\bar{C}^{-1}$ keeps the regions of convergence unchanged (Figure 5).



a) $\alpha = 0$ b) $\alpha = 0.5$ c) $\alpha = 0.7$

Figure 5. Regions of convergence for $\|\mathbf{f}_1\|$ and $\|\mathbf{f}_2\|$ (consider only for $\|\mathbf{f}_i\| \geq 0$).

This result implies a equalizer setting $\|\mathbf{f}_1\| = 1$ and $\|\mathbf{f}_2\| = 0$ is still valid (in it's convergence to s_1) for channels satisfying above assumptions. Furthermore, from the behavior of $\frac{1}{1-\alpha}$ as $\alpha \to 1$, the FS-CMA CPI equalizer under these channel assumption shows remarkable robustness of bit error rate (BER) performace when the cross-pole diversity (the condition number of matrix $A - B$ in Asumption 3) is degraded: Notice that in Figure 6 the BER is uniformly under 10^{-5} until about 10dB cross power ratio.

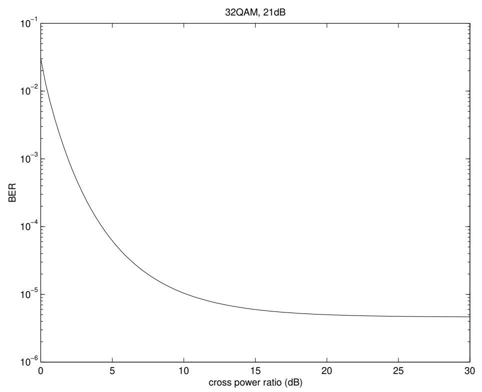


Figure 6. BER and cross power ratio α (32 QAM at 21 dB noise) from (31)

Since our analysis was performed under the assumption of a noise-free channel, the above plot is not precise. However, the experimental result in [12] shows exactly the same trend of Figure 6, because the effect of mild noise on the CMA cost function can be understood as "smoothing" the cost surface.

5 Conclusion

We have studied CPI equalizers of a factionally spaced system focusing on the blind FS-CMA CPI equalizer. We have shown that the solutions of the FS-CMA CPI equalizer match the ZF-solutions in the absence of noise. The FS-CMA CPI equalizer inherits the robust properties of conventional CMA equalizer (e.g. with noise and undermodelling). BER performance and initialization sensitivity have been shown to be robust to lack of corss-pole diversity for a simple channel class. Study of initialization for more general channels have yet to be studied.

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