# Approximate Channel Identification via $\delta$-Signed Correlation

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*Abstract* — **A method of approximate channel identification is proposed that is based on a simplification of the correlation estimator. Despite the numerical simplification (no multiplications or additions are required, only comparisons and an accumulator), the performance of the proposed estimator is not significantly worse than that of the standard correlation estimator. A free (user selectable) parameter moves "smoothly" from a situation with a small sum-squared channel estimation error but hard-to-identify channel peaks, to one with a larger sum-squared estimation error but easy-to-identify channel peaks. The proposed estimator is shown to be biased and its behavior is analyzed in a number of situations. Applications of the proposed estimator to sparsity identification, symbol timing recovery, and to the initialization of blind equalizers are suggested.**

## I. Introduction

Many communication channels have a naturally sparse structure (e.g., underwater acoustic communications, wireless communications in a hilly environment) and the channel impulse response has most of its energy concentrated in a few locations [1]. In the digital domain, the sampled channel response has only a few significant tap weights and most of the other tap weights are very small. Performing a channel identification or a channel equalization on such a sparse channel requires filters with long time spans relative to the number of nonzero tap weights. The advantages of exploiting sparsity are that less hardware is needed, fewer computations are required in the adaptation of the channel/equalizer coefficients, and there is less misadjustment noise (excess MSE) when an adaptive algorithm is used to track a smaller number of channel/equalizer coefficients [2]. In addition, using fewer taps tends to increase the speed of convergence of the adaptive algorithm [2]. To exploit channel sparsity, it is necessary to know the locations (and possibly the magnitudes) of the taps with significant energy.

The correlation method of identification [3], FFT-based approaches [4], and Least Squares channel estimation [5](and the recursive implementations such as RLS) are all designed to estimate the complete impulse response of the channel, and cannot be readily simplified to identify just the largest taps of the channel, unless the location of the taps is known *a priori*. Because of this, these methods are computationally complex, requiring at least O($N$) computations per channel tap, where $N$ is the length of the training sequence.

This paper proposes a method of estimating the channel impulse response by correlating the training sequence with the received data using the $\mathrm{sgn}_\delta$ function (a signum function with a $\delta$ sized 'dead zone'). This is a low complexity scheme that addresses the problem of sparsity identification and peak detection and may be used in a variety of applications. Though the estimates are biased, the estimation error is not significantly worse than when using the standard correlation estimator. In addition, the peaks or centers of energy of the channel taps may be more prominently displayed in some of the $\mathrm{sgn}_\delta$ versions than in the standard correlation estimator.

This paper is organized as follows. The standard correlation estimator is reviewed in Section II and the $\delta$-signed correlation estimator is introduced in Section III. The mean of the $\delta$-signed correlation estimator is derived in Section IV. Section V analyzes the behavior of the $\delta$-signed correlation estimator in a number of situations. Simulation results illustrating some of the properties of the proposed estimator are provided in Section VI. Possible applications of the proposed method are detailed in Section VII and Section VIII concludes.

## II. Background: The Correlation Method

The correlation method is suitable when the training sequence is self-orthogonal, i.e., white. The locations of the nonzero portions of the channel response can be determined by correlating the training sequence with the received data for all possible time shifts over a time window that spans the entire duration of the channel impulse response. The peaks of the correlation estimate give an idea of the time delays at which the channel shows significant energy.

The system model consists of the transmission of a source sequence $u(k)$ through a linear finite-impulse-response (FIR) channel. Let $r(k)$ denote the noisy output of the system represented by the impulse response $h(i)$. Let $L$ be the number

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of channel impulse response coefficients. Then the received sequence $r(k)$ can be expressed as

$$r(k) = \sum_{i=0}^{L-1} u(k-i)h(i) + w(k) \ , \tag{1}$$

where $w(k)$ is the additive noise sequence at the receiver. The "correlation estimates" $\hat{h}(i)$ of the channel taps can be calculated as

$$\hat{h}(i) = \frac{1}{N} \sum_{k=1}^{N} u(k)r(k+i) \ , \tag{2}$$

where $N$ is the length of the training sequence. It is well known that under suitable assumptions, namely a self-orthogonal training sequence and a zero-mean uncorrelated noise sequence, this method leads to unbiased estimates of the channel coefficients, with a variance that decays smoothly as the number of points used in the correlation grows.

## III. The $\delta$-Signed Correlation Method

As is clear from equation (2), the correlation method requires $N$ multiplications and $N$ additions for estimating each coefficient of the channel impulse response, leading to an O($N$) algorithm. This is computationally prohibitive in many applications, especially in systems operating at high data rates.

The kernel of the idea presented here is to observe that the most important information (in terms of the cross-correlation) is not really contained in the amplitudes of the data, but in its sign. Hence one can consider using a "signed correlation". Formally, consider estimates of the channel impulse response

$$\hat{h}_{\text{sgn}}(i) = \frac{1}{N} \sum_{k=1}^{N} \text{sgn}\{u(k)\} \ \text{sgn}\{r(k+i)\} \ , \tag{3}$$

where the signum or sign function is defined by

$$\text{sgn}(x) = \begin{cases} 1 & \text{if} \quad x > 0 \\ 0 & \text{if} \quad x = 0 \\ -1 & \text{if} \quad x < 0 \end{cases} .$$

In terms of computational complexity, equation (3) requires no multiplies, but simply a counter that accumulates how many times the signs agree and disagree. Obviously, $\hat{h}_{\text{sgn}}(i)$ differs from $\hat{h}(i)$, but are the differences significant?

This depends on the application. In [6], a variety of autocorrelation methods are considered for use in spectroscopy and radio astronomy. Under the assumption that the inputs are Gaussian, the true autocorrelation function can be inferred from that of a signed version, though the number of symbols necessarily increases for a desired level of accuracy. In communication systems, [7] has studied the performance degradation caused by the use of a signed correlator in the detection problem and [8, 9] have investigated the application of quantized correlation for frame synchronization in mobile OFDM systems.

The following intuitive argument suggests how the signed channel estimator might be improved. The most reliable data, from the point of view of the sign function, is that with a large absolute value, since even small amounts of noise may change the sign of the data that has a small absolute value. Consequently, it might be possible to improve the estimates by using only those data points that are above some critical threshold $\delta$. Now, consider the estimates

$$\hat{h}_{\delta}(i) = \frac{1}{N} \sum_{k=0}^{N-1} \text{sgn}\{u(k)\} \ \text{sgn}_{\delta}\{r(k+i)\} \ , \tag{4}$$

where the function $\text{sgn}_{\delta}$ is defined by

$$\text{sgn}_{\delta}(x) = \begin{cases} 1 & \text{if} \quad x > \delta \\ 0 & \text{if} \quad -\delta \le x \le \delta \\ -1 & \text{if} \quad x < -\delta \end{cases} .$$

Clearly, the signed-correlator is a special case of the $\text{sgn}_{\delta}$ correlator with $\delta = 0$.

## IV. Mean of the Estimators

This section derives expressions for the mean of the channel estimates for the signed and $\delta$-signed correlator under the following simplifying assumptions.

- The noise sequence $w(k)$ is zero mean, real, white and Gaussian with variance $\sigma^2$.
- The training symbols, $u(k)$ are i.i.d, uncorrelated with the noise sequence $w(k)$ and are selected from a BPSK constellation.

Since the training sequence is BPSK, equation (4) can be rewritten as

$$\hat{h}_{\delta}(i) = \frac{1}{N} \sum_{k=1}^{N} u(k)\text{sgn}_{\delta}\{r(k+i)\}. \tag{5}$$

Recall from equation (1)

$$r(k+i) = \sum_{m=0}^{L-1} h(m)u(k+i-m) + w(k+i). \tag{6}$$

Define the sequence $y_i(k)$ as

$$y_i(k) \triangleq \sum_{m=0, m\neq i}^{L-1} h(m)u(k+i-m) \tag{7}$$

and the vectors $\mathbf{b}_{-i}$ and $\mathbf{h}_{-i}$, each of length $L-1$ as

$$\mathbf{b}_{-i} \triangleq [b_0, \ \cdots \ , \ b_{i-1}, \ b_{i+1}, \ \cdots \ , \ b_{L-1}]^T \ ,$$
$$\mathbf{h}_{-i} \triangleq [h(0), \ \cdots \ , \ h(i-1), \ h(i+1), \ \cdots \ , \ h(L-1)]^T.$$

The sequence $y_i(k)$ is a weighted sum of i.i.d binary random variables and has a discrete probability distribution that takes on a value of $2^{1-L}$ at each possible $\mathbf{b}_{-i}^T\mathbf{h}_{-i}$, where the vector $\mathbf{b}_{-i} \in \{\pm 1\}^{L-1}$. Since $w(k+i)$ is independent of $y_i(k)$, the probability density function of $\{y_i(k) + w(k+i)\}$ can be computed by convolving the probability densities of $y_i(k)$ and $w(k+i)$. Hence, $\text{sgn}_\delta\{r(k+i)\}$ can be written as

$$\text{sgn}_\delta\{r(k+i)\} = \begin{cases} -u(k)\text{sgn}\{h(i)\} & \text{with prob } p(i) \\ 0 & \text{with prob } q(i)-p(i) \\ u(k)\text{sgn}\{h(i)\} & \text{with prob } 1-q(i) \end{cases}$$

with

$$p(i) = \sum_{\mathbf{b}_{-i}\in\{\pm 1\}^{L-1}} 2^{-L+1}\mathcal{Q}\left\{\frac{|h(i)| + \mathbf{b}_{-i}^T\mathbf{h}_{-i} + \delta}{\sigma}\right\}, \tag{8}$$

$$q(i) = \sum_{\mathbf{b}_{-i}\in\{\pm 1\}^{L-1}} 2^{-L+1}\mathcal{Q}\left\{\frac{|h(i)| + \mathbf{b}_{-i}^T\mathbf{h}_{-i} - \delta}{\sigma}\right\}, \tag{9}$$

where $\mathcal{Q}$ is the error function defined as

$$\mathcal{Q}(x) = \frac{1}{\sqrt{(2\pi)}}\int_x^\infty \exp\left(-\frac{y^2}{2}\right)dy. \tag{10}$$

Hence, the mean of the channel estimates for the $\delta$-signed correlator is given by,

$$E[\hat{h}_\delta(i)] = \text{sgn}\{h(i)\} - \text{sgn}\{h(i)\} \cdot \sum_{\substack{\mathbf{b}_{-i}\in\{\pm 1\}^{L-1} \\ b_L\in\{\pm 1\}}} 2^{-L+1}\mathcal{Q}\left\{\frac{|h(i)| + \mathbf{b}_{-i}^T\mathbf{h}_{-i} + b_L\delta}{\sigma}\right\}. \tag{11}$$

The mean of the $\delta$-signed correlation channel estimator is upper-bounded by unity, and hence the estimator is biased.

**Theorem 1 (Order Preservation)** *The $\delta$-signed correlation channel estimator preserves the order of the channel coefficients in the mean, i.e, if $|h(m_0)| \geq \cdots \geq |h(m_{L-1})|$ then $|E[\hat{h}_\delta(m_0)]| \geq \cdots \geq |E[\hat{h}_\delta(m_{L-1})]|$.*

Thus, on average, the tap estimates from the $\delta$-signed correlation are ranked in the same order as the taps of the channel impulse response. Since the signed-correlation channel estimator is a special case of the $\delta$-signed correlator, it also satisfies the order preservation property.

### A. *Gaussian Approximation*

If the length of the channel impulse response $L$ is large it is possible to employ the central limit theorem and approximate the probability density function of $\{y_i(k) + w(k+i)\}$ (recall equation (7)) as a zero-mean Gaussian distribution. Formally, this requires the satisfaction of the Lindeberg-Feller condition (see [10]). Let us assume that the Gaussian approximation holds, and that the channel is unit norm, i.e., $\|\mathbf{h}\|^2 = 1$. Then,

$$[y_i(k) + w(k+i)] \sim \mathcal{N}(0, \sigma_i^2) \tag{12}$$

where $\sigma_i^2 = 1 - |h(i)|^2 + \sigma^2$. Under this approximation, equations (8) and (9) simplify to

$$p(i) = \mathcal{Q}\left\{\frac{|h(i)| + \delta}{\sigma_i}\right\} \; ; \quad q(i) = \mathcal{Q}\left\{\frac{|h(i)| - \delta}{\sigma_i}\right\} \tag{13}$$

and the mean of the $\delta$-signed correlator simplifies to

$$E[\hat{h}_\delta(i)] = \text{sgn}\{h(i)\}\left[1 - \mathcal{Q}\left\{\frac{|h(i)| + \delta}{\sqrt{1-|h(i)|^2+\sigma^2}}\right\} - \mathcal{Q}\left\{\frac{|h(i)| - \delta}{\sqrt{1-|h(i)|^2+\sigma^2}}\right\}\right]. \tag{14}$$

## V. Asymptotic Analysis

In an effort to understand the behavior of the signed and $\delta$-signed correlation estimators, this section considers the special cases when $\sigma \to 0$ and $\sigma \to \infty$. Since the signed and $\delta$-signed correlation estimators are typically biased, we focus attention on the relative magnitudes of the channel tap coefficients.

### A. *2-Tap Channel*

Consider a channel impulse response with only two taps. Without loss of generality, we assume that $|h(0)| \geq |h(1)|$. Define the ratio of the magnitudes of the tap coefficients, $\Gamma$ and the ratio of the magnitudes of the mean of the estimates $\hat{\Gamma}_\delta$ as,

$$\Gamma \triangleq \frac{|h(0)|}{|h(1)|} \geq 1 \quad \text{and} \quad \hat{\Gamma}_\delta \triangleq \frac{|E[\hat{h}_\delta(0)]|}{|E[\hat{h}_\delta(1)]|}. \tag{15}$$

By using the approximation

$$\mathcal{Q}(x) \approx \frac{1}{2} - \frac{x}{\sqrt{2\pi}}, \qquad \text{if } x \ll 1 \tag{16}$$

on equation (11), it can be shown that

$$\lim_{\sigma\to\infty} |E[\hat{h}_\delta(i)]| = \frac{2|h(i)|}{\sqrt{2\pi}\sigma}, \qquad \forall\, i = 1, 2 \tag{17}$$

and hence

$$\lim_{\sigma\to\infty} \hat{\Gamma}_\delta = \Gamma. \tag{18}$$

In the noiseless scenario (when $\sigma \to 0$) recall that

$$\lim_{\sigma\to 0} \mathcal{Q}\left(\frac{x}{\sigma}\right) = \begin{cases} 1 & \text{if } x < 0 \\ 0.5 & \text{if } x = 0 \\ 0 & \text{if } x > 0 \end{cases}, \tag{19}$$

and hence

$$\lim_{\sigma\to 0}\hat{\Gamma}_\delta = \begin{cases} \infty & \text{if} \quad \delta < |h(0)| - |h(1)| \\ 3 & \text{if} \quad \delta = |h(0)| - |h(1)| \\ 1 & \text{if} \quad \delta > |h(0)| - |h(1)| \end{cases}. \tag{20}$$

For example, consider a 2-tap channel with an impulse response of $\mathbf{h} = [0.8,\ 0.6]$. Figure 1 plots the ratio of the magnitudes of the mean of the $\delta$-signed correlation estimator, as a function of the SNR, for various values of the threshold $\delta$. From the plot, it is clear that at high values of SNR, the tap with the larger magnitude is greatly enhanced as long as the value of the threshold $\delta \leq |h(0)| - |h(1)|$. Such a property of the $\delta$-signed estimator would be useful in an application where only the location of cursor is required.

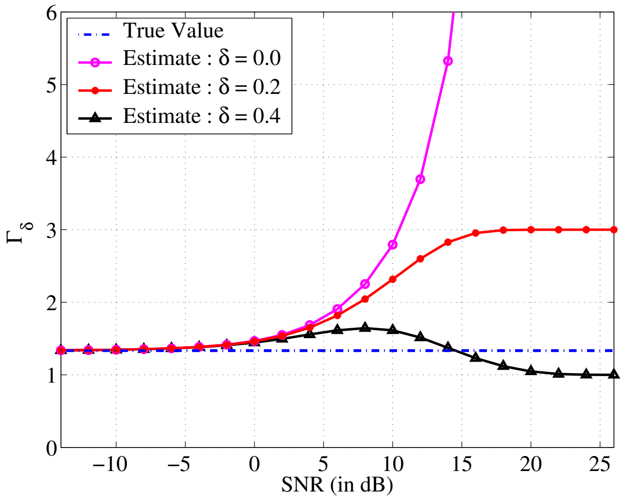


Fig. 1: The ratio of the magnitudes of the mean of the $\delta$-signed correlation estimator for a 2-tap channel, $\mathbf{h} = [0.8,\ 0.6]$ as a function of the SNR for various choices of $\delta$.

## B. *L-Tap Channel*

Now consider a channel impulse response with $L$ taps. Let us assume that the Gaussian approximation of Section IV.A holds. Using the approximation in equation (16) on the mean expression of equation (14) we have,

$$\lim_{\sigma\to\infty}\frac{|E[\hat{h}_\delta(i)]|}{|E[\hat{h}_\delta(j)]|} = \frac{|h(i)|}{|h(j)|}. \tag{21}$$

Hence, at low values of SNR, the $\delta$-signed correlation estimator preserves the relative magnitudes of the channel taps in the mean.

**Theorem 2 (Dominant Tap Enhancement)** *If the channel impulse response coefficients are such that $|h(i)| \geq |h(j)|$ for some pair of integers $(i, j)$ and $|h(i)| \leq \sqrt{2(1+\sigma^2)/3}$, then the signed correlation channel tap estimate of the L-tap channel, under the Gaussian approximation, satisfies*

$$\hat{\Gamma}_{ij}^{sgn} \triangleq \frac{|E[\hat{h}_{sgn}(i)]|}{|E[\hat{h}_{sgn}(j)]|} \geq \frac{|h(i)|}{|h(j)|} \triangleq \Gamma_{ij} \tag{22}$$

Hence, the dominant tap is augmented compared to the smaller taps, which reinforces the observations from the example in figures 4 and 5 that the taps with the largest energy are more clearly visible in the simpler $\delta$-signed correlator than in the standard correlator. Note that this property of the $\delta$-signed correlation estimator makes it suitable for applications like sparsity detection, where the aim is to determine the locations of the significant taps of the channel impulse response.

# VI. Simulation Results

The performance of the signed and $\delta$-signed correlation estimators were studied using simulations. The channel impulse response (see Figure 2) of a measured microwave channel [11] was chosen for the simulations. The data set was drawn from a BPSK source constellation. A 511-length PN sequence, as defined by the High Definition Television (HDTV) standards [12], was chosen as the training data. The SNR at the receiver was assumed to be 15 dB. Since the $\delta$-signed channel estimator is biased, the estimated channel impulse response was normalized to unit norm, i.e., $||\hat{\mathbf{h}}_\delta||^2 = 1$.

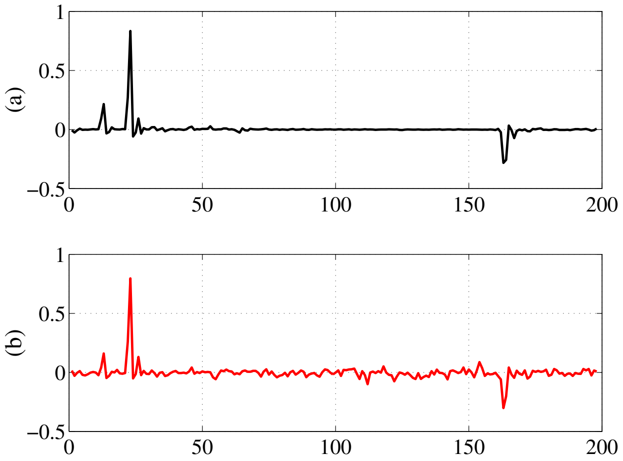


Fig. 2: (a) Channel impulse response of a measured channel from the SPIB database [11] ; (b) Correlation estimate of the impulse response.

Figure 3 compares the average sum-squared channel estimation error of the $\delta$-signed estimator, for various choices of $\delta$, with the performance of the standard correlation estimator. The abscissa of “-1” in the figure corresponds to the correlation estimator, while abscissas between “0” and “9” represents the estimation error as $\delta$ varies from 0 to $1.8\lambda$ in steps of $0.2\lambda$ each. The value of $\lambda$ is chosen to be the

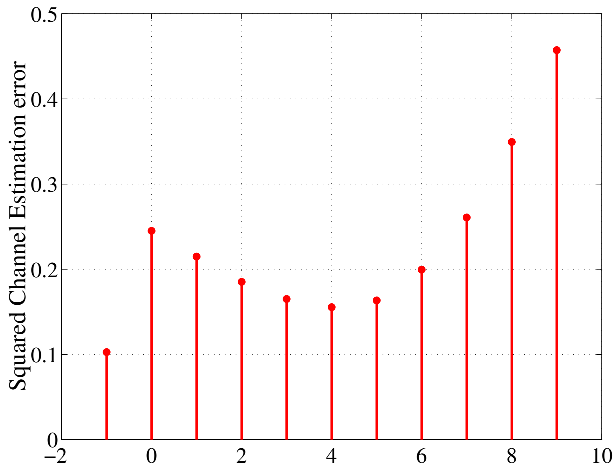


Fig. 3: Averaged sum-squared channel estimation error as a function of $\delta$. The abscissa "-1" corresponds to the standard correlation estimator. The abscissas "0" to "9" correspond to $\delta$-signed estimators as $\delta$ takes on values between 0 to $1.8\lambda$ in steps of $0.2\lambda$ each.

mean of the absolute value of the received signal vector, i.e., $\lambda = E[|r(k)|]$. Although the performance of the $\delta$-signed correlator is worse than the standard correlation estimator, the degradation is small and is only on the order of a couple of dB. Furthermore, the performance of the $\delta$-signed estimator improves with increasing values of $\delta$ up to a point and this suggests that there exists some optimal value for the choice of $\delta$. For this particular example, $0.8\lambda$ seems to be the optimal choice of $\delta$. However, the value of the optimal $\delta$ may be a function of the channel and hence difficult to predict.

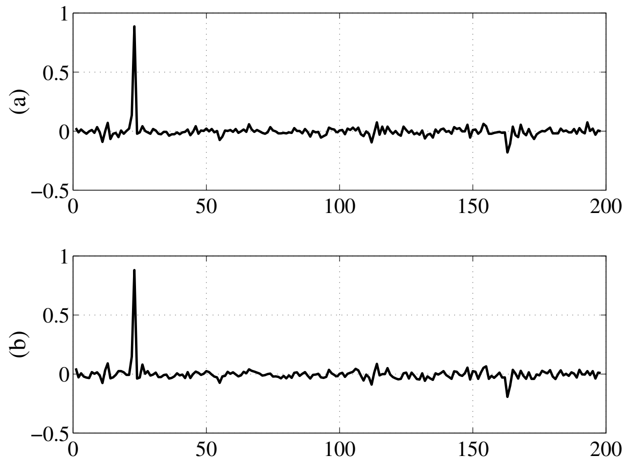


Fig. 4: The estimated channel impulse response using the signed correlation method in (a), and using the $\delta$-signed correlation method for a value of $\delta = 0.4\lambda$ in (b)

Figures 4 and 5 show the estimates of the channel in Figure

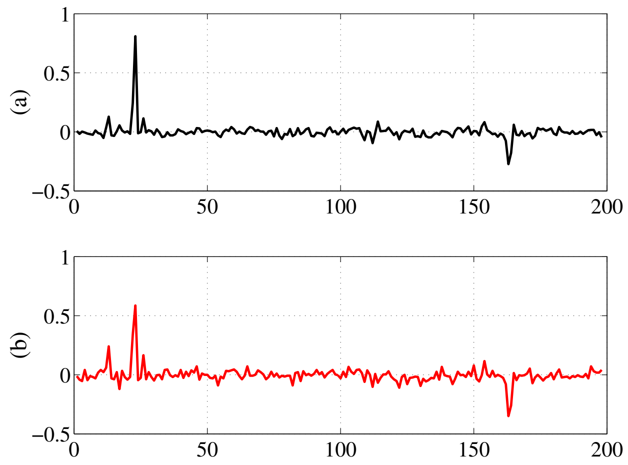


Fig. 5: The estimated channel impulse response using $\delta$-signed correlation method for $\delta$ values of $0.8\lambda$ in (a) and $1.4\lambda$ in (b).

2 for a number of different values of $\delta$. Figure 4 shows the channel estimates for $\delta$ values of 0 in (a) and $0.4\lambda$ in (b). Figure 5 continues with $0.8\lambda$ in (a) and $1.4\lambda$ in (b). The signed version ($\delta = 0$) clearly shows the location of the cursor but much of the channel detail is lost, due to the coarseness of the sign function. As $\delta$ grows, the three peaks of the channel become more and more prominent. Especially for the choice of $\delta = 1.4\lambda$, although the channel estimates are very noisy, the three peaks of the channel are quite prominent (see Figure 5) and hence suitable for sparsity detection. For yet larger values of $\delta$ (not shown), there are not enough nonzero points to register and the estimates in equation (4) degrade. This example suggests that careful selection of $\delta$ may allow the simpler estimator to retain certain details of the channel despite the numerical simplifications.

## VII. Using These Rough Estimates

The approximate channel estimates given by the $\delta$-signed correlator can be used to initialize a blind adaptive equalizer (or a decision feedback equalizer, if the estimates are accurate enough). The taps that follow the cursor (generally, the largest peak) can be used directly to initialize (for instance) a blind infinite-impulse-response (IIR) equalizer. As shown in [13] if this initialization is good enough to reduce the SINR to about -3.8 dB, it can be guaranteed that the constant modulus algorithm (CMA) will converge to a minimum corresponding to a delay consistent with the initialization. In particular, this guarantees that no undesirable saddle points will be encountered.

The rough channel estimates provided by the $\delta$-signed correlator can be used to detect the presence of sparsity in the channel. By setting an appropriate threshold, all taps below this threshold may be assumed insignificant (set to zero). Exploiting this sparsity is as simple as initializing and adapting only those parameters in or around these nonzero regions.

When sparse adaptation is being done in the equalizer, typically only a subset of the taps are actually adapted. One problem, however, is that in a time varying situation the locations of the significant channel taps may change. A low complexity solution to this problem is to use the $\delta$-signed channel estimator. One can adapt the sparse equalizer as before – but when the rough channel estimates indicate energy at a location where there are no taps in the equalizer, then these taps can begin to be adapted.

Finally, some kind of correlation is often done to look for the start of each frame. Monitoring the time difference between successive peaks in the correlation (i.e., between successive frames) and dividing by the number of symbols expected in that frame, gives a measure of clock timing. For example, [14] describes a timing recovery technique based on detecting the "field sync" signals in the HDTV data record and using some sort of phase locked loop (PLL). As illustrated in Section VI the $\delta$-signed correlation estimator is especially well suited for detecting the peaks and hence can be used for timing recovery.

## VIII. Summary

This paper has proposed a method of correlating the training sequence with the received data using the $\text{sgn}_\delta$ function (a signum function with a $\delta$ sized 'dead zone'). The $\delta$-signed correlation estimator was shown to be biased and its behavior was analyzed in a number of situations. The performance of the $\delta$-signed correlation estimator was found to be only marginally worse than that of the correlation estimator. It was further shown that the $\delta$-signed correlation estimator displays the peaks in the channel impulse response more prominently than the correlation estimator, for suitable choices of $\delta$. This property of the $\delta$-signed correlation estimator makes it suitable for applications like peak detection and sparsity identification. In scenarios where the use of a correlation procedure would be useful, but where computational complexity is of concern, it may be worthwhile to consider using a $\delta$-signed correlator.

## References


[1] J. R. Treichler and M. G. Larimore, "Thinned impulse responses for adaptive FIR filters, *Proc. IEEE Int. Conf. Acoustics, Speech and Signal Processing*, pp. 631-634, May 1982.

[2] B. Widrow, J. M. McCool, M. G. Larimore and C. R. Johnson, Jr., "Stationary and nonstationary learning characteristics of the LMS adaptive filter," *Proc. IEEE*, Vol. 64, pp. 1151-1162, Aug. 1976.

[3] W. W. Lichtenberger, "A technique of linear system identification using correlating filters," *IRE Trans. on Automatic Control*, Vol. AC-6, No. 3, pp. 183-199, 1961.

[4] W. E. Butcher and G. E. Cook, "Identification of linear sampled data system by transform techniques," *IEEE Trans. on Automatic Control*, Vol. AC-14, pp. 582-584, 1969.

[5] S. Haykin, "*Adaptive Filter Theory*," Prentice-Hall, Upper Saddle River, NJ, 1996.

[6] J. B. Hagen and D. T. Farley, "Digital-correlation techniques in radio science," *Radio Science*, Vol. 8, No. 8-9, pp. 775-784, Aug.-Sept. 1973.

[7] P. Willett and P. F. Swaszek, "On the performance degradation from one-bit quantized detection," *IEEE Trans. on Information Theory*, Nov 1995.

[8] J. J. van de Beek, M. Sandell, M. Isaksson and P. Ola Borjesson, "Low-complex frame synchronization in OFDM systems," *Proc. IEEE Int. Conf. Universal Personal Communications*, Toronto, Canada, pp. 982-986, Sep. 1995.

[9] Meng-Han Hsieh and Che-Ho Wei, "A low-complexity frame synchronization and frequency offset compensation scheme for OFDM systems over fading channels," *IEEE Trans. on Vehicular Technology*, Vol. 48, No. 5, pp. 1596-1609, Sept. 1999.

[10] S. I. Resnick, "*A Probability Path*," Birkhäuser Boston, 1999.

[11] Microwave channel models from the Signal Processing Information Base (SPIB), *http://spib.rice.edu/spib/microwave.html*.

[12] "ATSC Standard A/54," *ATSC Digital Television Standard*, Oct. 1995.

[13] P. Schniter and C. R. Johnson, Jr., "Sufficient Conditions for the Local Convergence of Constant Modulus Algorithms," IEEE Trans. on Signal Processing, Vol.48, No. 10, pp. 2785-2796, Oct. 2000

[14] Kibum Kim, Hyunsoo Shin and Dongil Song, "A symbol timing recovery using the segment sync data for the digital HDTV GA VSB system," *IEEE Trans. on Consumer Electronics*, Vol. 42, No. 3, pp. 651-656, Aug 1996.