Time-Reversal Diversity in Decision Feedback Equalization*

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Abstract

This paper considers the use of a time-reversal decision feedback equalizer (DFE) in conjunction with a conventional DFE for block processing of the received signal. A receiver structure that combines the conventional DFE outputs and the time-reversal DFE outputs is proposed. An optimal combining scheme of the two streams that minimizes the mean-squared error (MSE) at the receiver output is derived. The inherent diversity between the soft outputs of the two streams is described and quantified under a few simplifying assumptions. For the special case of a symmetric channel, the optimal DFE filter setting that minimizes the MSE at the output of the diversity combining block is derived. Simulation results demonstrating performance gains for the proposed receiver structure are presented for both symmetric and asymmetric channels.

1 Introduction

Future generation wireless communication technology promises an increase in data rate capabilities of present radio links. As we go for higher rates the frequency distortion caused by the channel becomes worse. The distortion caused by the resulting overlap of received symbols is called intersymbol interference (ISI), which, if left uncompensated, causes high error rates. One suboptimal scheme that is widely used in practice, due to its simple implementation, is the linear equalizer. But, in suppressing the intersymbol interference, the linear equalizer inevitably enhances the noise. This basic limitation of a linear equalizer's ability to cope with severe ISI has motivated a considerable amount of research into suboptimal nonlinear equalizers with low computational complexity such as the decision feedback equalizer (DFE).

The idea of using previous decisions to cope with the ISI problem was first introduced in 1967 by Austin [1]. The decision feedback receiver that minimizes the mean-squared error (MSE) between the input to the threshold device and the transmitted symbol was first obtained by Monsen [2]. The action of the DFE is to feed back a weighted sum of the past decisions, assumed to be correct, to cancel the ISI they cause in the present signal interval. However, when incorrect decisions are fed back through the feedback filter of a DFE, it induces a burst

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of errors causing performance degradation and this phenomenon is known as error propagation.

Bidirectional equalization of the received signal was first considered in [3, 4]. The use of a selective time-reversal DFE structure was described in [5]. A time-reversal operation is done by reversing the sequential order of the received samples, in time, prior to equalization. As a result, the equivalent channel impulse response as seen by the equalizer becomes a time-reverse of the actual channel impulse response. This results in the minimum phase roots of the channel becoming the maximum phase roots and vice-versa. When a finite length DFE is used, the performance is usually different for the forward DFE and the time-reversal DFE and hence selecting the stream with lesser MSE is beneficial. However, the performance can be further improved by combining the two streams rather than selecting one.

This approach has been used in a recent publication [7], in which the authors, based on a reconstruction and arbitration technique, exploit the diversity in error propagation between the forward DFE outputs and the time-reversal DFE outputs to improve performance. Error propagation is causal and hence the error burst for a conventional DFE and a time-reversal DFE proceed in opposite directions in time. This results in a low correlation for the errors caused by the forward DFE and the time-reversal DFE. Exploiting this diversity has been the motivation behind [7].

In this paper, the use of a DFE with a time-reversal structure in conjunction with a conventional DFE is considered. Furthermore, it is shown that the soft outputs of a forward DFE and the time-reversal DFE possess an inherent diversity. The term diversity is used here to refer to the low correlation in the error terms of the soft outputs. The diversity is a result of the nonlinear structure of the DFE and the fact that we allow anti-causal processing of the received signal and is present even in the absence of error propagation. This diversity, when exploited, results in a decrease in the noise gain term and the residual ISI component. A receiver structure that combines the conventional DFE output and the time-reversal DFE output, to exploit this inherent diversity between the two streams, is proposed and the diversity that can be potentially exploited is quantified.

The organization of the paper is as follows. Section 2 describes the system model and states the assumptions made in this paper. An optimal combining scheme for the two streams that minimizes the MSE at the output of the receiver structure is derived in Section 3. Section 4 describes the time-reversal diversity inherent in the noise gain term and the residual ISI component for a symmetric channel. In Section 5, the optimal unbiased DFE tap setting that minimizes the MSE at the output of the diversity combining scheme is derived for a symmetric channel. Simulation results demonstrating performance gains of the proposed receiver structure are provided in Section 6 for both symmetric and asymmetric channels. A summary of the contributions of this paper and a brief discussion of possible directions for future work is provided in Section 7.

2 System Model

The block diagram of the proposed receiver structure is shown in Figure 1. The source symbols $s(n)$ are transmitted though a frequency selective channel. The channel incorporates the effects of frequency selective distortion caused by the physical medium and transmit pulse shaping and is modeled as a linear FIR filter $\mathbf{c}$. The transmitted signal is further corrupted by additive white

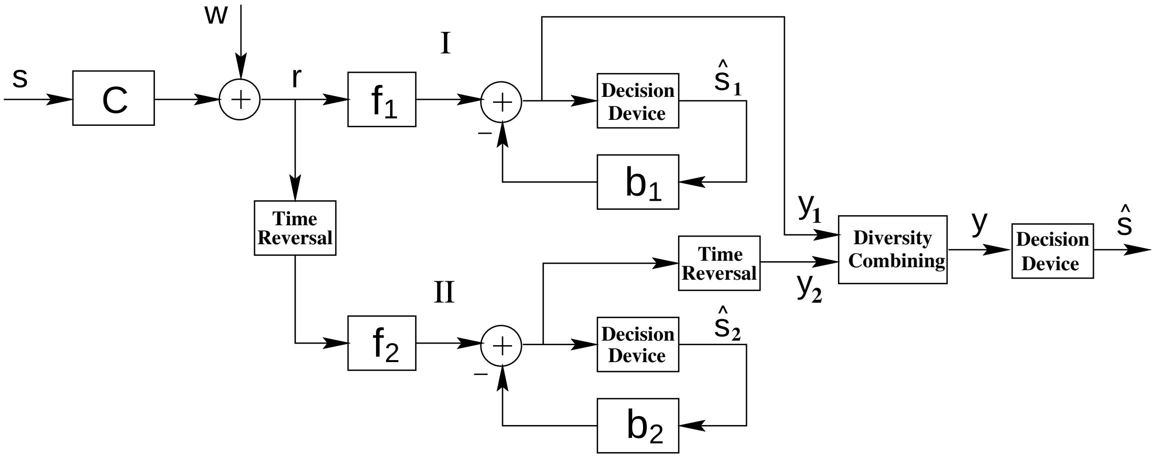

Figure 1: Time-Reversal DFE structure

noise at the receiver. The received signal $r(n)$ is given by,

$$r(n) = \sum_{k=0}^{L_c-1} c(k)s(n-k) + w(n), \quad n = 1, 2, \cdots, N \tag{1}$$

where $\mathbf{c} = [c(0)\ c(1)\ \ldots\ c(L_c-1)]^T$ is the channel impulse response with L_c taps and w(n) is the noise term.

The receiver structure of Figure 1 processes a data block of size N using two different streams. The received signal $r(n)$ is equalized in stream I using a DFE with an inherent detection delay $\delta 1$. The feedforward filter $\mathbf{f_1}$ has L_{f1} taps and its impulse response is denoted by the vector $\mathbf{f_1} = [f_1(0)\ f_1(1)\ \ldots\ f_1(L_{f1}-1)]^T$ while the feedback filter $\mathbf{b_1}$ has L_{b1} taps and an impulse response $\mathbf{b_1} = [b_1(1)\ b_1(2)\ \ldots\ b_1(L_{b1})]^T$. The combined channel-feedforward filter impulse response for stream I is given by $\mathbf{h_1} = \mathbf{c} * \mathbf{f_1}$, where $*$ denotes the convolution operation. $h_1(\delta 1)$ is then referred to as the cursor.

The received signal $r(n)$ is time reversed in stream II. The time-reversed version of the received signal can then be expressed as

$$\begin{aligned}\tilde{r}(n) &= r(N-n+1) \\ &= \sum_{k=-(L_c-1)}^{0} c(-k)\tilde{s}(n-k) + \tilde{w}(n)\end{aligned} \tag{2}$$

where tilde implies time-reversal, $\tilde{s}(n) = s(N-n+1)$ is the time-reversed source sequence and $\tilde{w}(n) = w(N-n+1)$ is the time-reversed noise sequence. This is equivalent to transmitting the time-reversed source sequence through a channel whose coefficients are the time-reversed version of $\mathbf{c}$. Let $\mathbf{f_2}$ with L_{f2} taps and $\mathbf{b_2}$ with L_{b2} taps be the feedforward and feedback filter settings of the DFE that equalizes the time-reversed channel $\tilde{\mathbf{c}} = [c(L_c-1)\ \cdots\ c(1)\ c(0)]^T$. The combined channel-feedforward filter impulse response for this stream is $\mathbf{h_2} = \tilde{\mathbf{c}} * \mathbf{f_2}$. Let $\delta 2$ be the delay introduced by the equalizer in stream II. Note that the optimum burst synchronization for the two streams may be different (see [5]), in general, as it depends on the detection delay of the equalizer. For a particular equalizer structure we may explicitly include a time shift by a multiple of the symbol duration in equation (2) to account for such a change in receiver timing for the two modes of operation.

In this paper we make the following assumptions.

- The FFF and the FBF for the two streams are those designed to generate the unbiased DFE settings, i.e., $h_1(\delta 1) = h_2(\delta 2) = 1$.
- The decisions that are fed back to the FBF of the DFE are correct, i.e., ideal feedback.
- The source sequence $s(n)$ is i.i.d and has unit variance.
- The noise sequence $w(n)$ is zero mean, white, has a variance σ_w^2 and is uncorrelated to the source sequence.

$y_1(n)$ are the soft inputs to the decision device of stream I and $y_2(n)$ are the time-reversed soft inputs to the decision device of stream II. The soft input sequences $y_1(n)$ and $y_2(n)$ can be expressed as,

$$y_1(n) = s(n) + \eta_1(n) \tag{3}$$

$$y_2(n) = s(n) + \eta_2(n) \tag{4}$$

where

$$\begin{aligned}\eta_1(n) = & \sum_{m=0}^{L_{f1}-1} f_1(m)w(n+\delta 1-m) + \sum_{m=0}^{\delta 1-1} h_1(m)s(n+\delta 1-m) \\ & + \sum_{m=\delta 1+L_{b1}+1}^{L_{f1}+L_c-2} h_1(m)s(n+\delta 1-m)\end{aligned} \tag{5}$$

$$\begin{aligned}\eta_2(n) = & \sum_{m=0}^{L_{f2}-1} f_2(m)w(n+L_c-1-\delta 2+m) + \sum_{m=0}^{\delta 2-1} h_2(m)s(n-\delta 2+m) \\ & + \sum_{m=\delta 2+L_{b2}+1}^{L_{f2}+L_c-2} h_2(m)s(n-\delta 2+m)\end{aligned} \tag{6}$$

Let σ_1^2 and σ_2^2 be the variances of $\eta_1(\cdot)$ and $\eta_2(\cdot)$ respectively. We use ρ to denote the coefficient of correlation between the sequences $\eta_1(\cdot)$ and $\eta_2(\cdot)$, i.e.,

$$\rho = \frac{\mathbf{E}[\eta_1\eta_2]}{\sigma_1\sigma_2} \tag{7}$$

where $\mathbf{E}[\cdot]$ is the expectation operation.

3 Diversity Combining

The soft output $y(n)$ of the diversity combining block in Figure 1 is generated as a linear combination of the sequences $y_1(n)$ and $y_2(n)$, i.e.,

$$\begin{aligned} y(n) &= \alpha y_1(n) + (1-\alpha)y_2(n) \\ &= s(n) + \alpha\eta_1(n) + (1-\alpha)\eta_2(n) \end{aligned} \tag{8}$$

The MSE for the soft output $y(n)$ is given by,

$$\begin{aligned} \text{MSE} &= \mathbf{E}[|y(n) - s(n)|^2] \\ &= \alpha^2\sigma_1^2 + (1-\alpha)^2\sigma_2^2 + 2\rho\alpha(1-\alpha)\sigma_1\sigma_2 \end{aligned} \tag{9}$$

The weighting factor α is optimized to minimize MSE and the optimal value of α is obtained by setting,

$$\frac{\partial \text{MSE}}{\partial \alpha} = 0 \tag{10}$$

$$\Rightarrow \qquad \alpha = \frac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} \tag{11}$$

When $\sigma_1 = \sigma_2$, the optimal weighting factor is $\alpha = \frac{1}{2}$ and this is known as equal-gain combining.

4 Time-Reversal Diversity

For a symmetric channel, the time-reversed channel impulse $\tilde{\mathbf{c}}$ is equal to the channel impulse response $\mathbf{c}$. Hence, the same DFE tap settings can be used for the two streams, i.e., $\mathbf{f_1} = \mathbf{f_2} = \mathbf{f}$ and $\mathbf{b_1} = \mathbf{b_2} = \mathbf{f}$. Further, the mean-squared error (MSE) for the two streams, σ_1 and σ_2, are equal and the optimal weighting factor of equation (11) is given by $\alpha = \frac{1}{2}$. The soft outputs, $y(n)$ of the diversity combining block can be expressed as,

$$y(n) = s(n) + \eta(n) \tag{12}$$

where $\eta(n)$ is the sum of the noise and the residual ISI components and,

$$\eta(n) = \frac{\eta_1(n) + \eta_2(n)}{2} \tag{13}$$

The combined channel-feedforward impulse response $\mathbf{h} = \mathbf{c} * \mathbf{f}$ is a L_h length vector, where $L_h = L_c + L_f - 1$. Let δ be the detection delay of the DFE. We define the vector $\mathbf{g}$,

$$g(k) = \begin{cases} 0, & \delta \le k \le \delta + L_b \\ h(k), & \text{otherwise} \end{cases} \tag{14}$$

Recall that L_b is the length of the feedback filter. Equation (13) can be rewritten as,

$$\begin{aligned} \eta(n) = \frac{1}{2}\sum_{k=0}^{L_h-1} g(k)\{s(n+\delta-k) + s(n-\delta+k)\} \\ + \frac{1}{2}\sum_{k=0}^{L_f-1} f(k)\{w(n+\delta-k) + w(n+L_c-1-\delta+k)\} \end{aligned} \tag{15}$$

Equation (15) expresses the MSE of the overall system as a sum of the noise gain and the residual ISI terms.

$$\text{Noise gain term} = \sum_{k=k_{min}}^{k_{max}} \left\{ \frac{f(k) + f(2\delta + 1 - L_c - k)}{2} \right\}^2 \sigma_w^2 \tag{16}$$

where

$$k_{min} = \min(0, 2\delta + 1 - L_h) \quad \text{and} \quad k_{max} = \max(L_f - 1, 2\delta + 1 - L_c) \tag{17}$$

and

$$\text{Residual ISI term} = \sum_{k=j_{min}}^{j_{max}} \left\{ \frac{g(k) + g(2\delta - k)}{2} \right\}^2 \tag{18}$$

where

$$j_{min} = \min(0, 2\delta + 1 - L_h) \quad \text{and} \quad j_{max} = \max(L_h - 1, 2\delta) \tag{19}$$

For each individual stream, the noise gain term and the residual ISI terms are $\mathbf{f}^T\mathbf{f}\sigma_w^2$ and $\mathbf{g}^T\mathbf{g}$ respectively. From equations (16) and (18), we observe that the equal-gain diversity combining scheme has decreased[1] the contributions of both the noise gain and the residual ISI terms on the effective MSE when compared to either the normal-mode or time-reversal mode operation of the DFE. This diversity is a result of the non-linear structure of the DFE and the fact that we allow anti-causal processing of the received signal. For example, the output of a linear equalizer under time-reversal does not exhibit any diversity.

5 Optimal DFE for Equal-Gain Combining

Optimizing the MMSE-DFE taps to minimize the MSE at the decision device of each individual stream would not necessarily minimize the effective MSE of the receiver structure that incorporates diversity combining. In this section the optimization of the unbiased MMSE-DFE taps to minimize $\mathbf{E}[\eta^2]$, the effective MSE, for a symmetric channel is described.

The DFE taps are optimized for a fixed system delay, δ. The channel convolution matrix for the channel $\mathbf{c}$ is an $L_h \times L_f$ matrix and is,

$$C = \begin{bmatrix} c(0) & 0 & \cdots & 0 \\ \vdots & c(0) & \cdots & \vdots \\ c(L_c - 1) & \vdots & \cdots & 0 \\ 0 & c(L_c - 1) & \cdots & c(0) \\ \vdots & 0 & \cdots & \vdots \\ 0 & \vdots & \cdots & c(L_c - 1) \end{bmatrix} \tag{20}$$

[1]This can be easily proved using the Cauchy-Schwartz inequality

The combined channel feedforward impulse response can be expressed as $\mathbf{h} = C\mathbf{f}$. The optimal feedback filter taps of the DFE should exactly cancel the L_b taps of $\mathbf{h}$ that follow the cursor h_δ to minimize MSE. We define the $L_h \times L_h$ matrix V as,

$$V = \begin{bmatrix} \mathbf{I}_{\delta+1\times\delta+1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0}_{L_b\times L_b} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \tag{21}$$

We define the vector $\mathbf{g}$ as

$$\mathbf{g} = VC\mathbf{f} - \mathbf{e}_\delta \tag{22}$$

where $\mathbf{e}_\delta$ is a column vector of length L_h and,

$$\mathbf{e}_\delta = [\mathbf{0}_{1\times\delta}\ 1\ \mathbf{0}_{1\times L_h-\delta-1}]^T \tag{23}$$

The variance of the residual ISI term in each of the streams is $\mathbf{g}^T\mathbf{g}$ and the combined residual ISI term of equation (18) can be expressed as

$$\text{Residual ISI term} = \frac{1}{4}(S_1+S_2)^T\mathbf{g}^T\mathbf{g}(S_1+S_2) \tag{24}$$

where S_1 and S_2 are $(2L_h-1)\times L_h$ matrices that perform shifting and time-reversal operations on the vector $\mathbf{g}$, prior to averaging. The matrices S_1 and S_2 are constructed as,

$$S_1 = \begin{bmatrix} \mathbf{0}_{L_h-1-\delta\times L_h} \\ \mathbf{I}_{L_h\times L_h} \\ \mathbf{0}_{\delta\times L_h} \end{bmatrix} \quad ; \quad S_2 = \begin{bmatrix} \mathbf{0}_{\delta\times L_h} \\ \tilde{\mathbf{I}}_{L_h\times L_h} \\ \mathbf{0}_{L_h-1-\delta\times L_h} \end{bmatrix} \tag{25}$$

where $\mathbf{0}$ refers to a zero matrix, $\mathbf{I}$ refers to an identity matrix and $\tilde{\mathbf{I}}$ is a time-reversal matrix, i.e., with it's anti-diagonal elements as unity and all the other elements as zero. Similarly, the noise gain term of equation (16) can be expressed as,

$$\text{Noise gain term} = \frac{\sigma_w^2}{4}(M_1+M_2)^T\mathbf{f}^T\mathbf{f}(M_1+M_2) \tag{26}$$

where M_1 and M_2 are $(L_h+L_f-1)\times L_f$ matrices that perform shifting and time-reversal operations on the vector $\mathbf{f}$ before averaging. The matrices M_1 and M_2 are constructed as shown below. If $2\delta+1-L_h \geq 0$ then,

$$M_1 = \begin{bmatrix} \mathbf{I}_{L_f\times L_f} \\ \mathbf{0}_{L_h-1\times L_f} \end{bmatrix} \quad ; \quad M_2 = \begin{bmatrix} \mathbf{0}_{2\delta+1-L_h\times L_f} \\ \tilde{\mathbf{I}}_{L_f\times L_f} \\ \mathbf{0}_{2L_h-2\delta-2\times L_f} \end{bmatrix} \tag{27}$$

and if $2\delta+1-L_h < 0$ then,

$$M_1 = \begin{bmatrix} \mathbf{0}_{L_h-2\delta-1\times L_f} \\ \mathbf{I}_{L_f\times L_f} \\ \mathbf{0}_{2\delta\times L_f} \end{bmatrix} \quad ; \quad M_2 = \begin{bmatrix} \tilde{\mathbf{I}}_{L_f\times L_f} \\ \mathbf{0}_{L_h-1\times L_f} \end{bmatrix} \tag{28}$$

The optimization of the DFE taps to minimize MSE is formulated as,

$$\min_{\mathbf{f}} \frac{1}{4}\Big[(S_1+S_2)^T(VC\mathbf{f}-\mathbf{e}_\delta)^T(VC\mathbf{f}-\mathbf{e}_\delta)(S_1+S_2) + (M_1+M_2)^T\mathbf{f}^T\mathbf{f}(M_1+M_2)\sigma_w^2\Big] \tag{29}$$

The solution to the optimization problem of equation (29) is,

$$\mathbf{f} = [(VC)^T(S_1+S_2)^T(S_1+S_2)VC + \sigma_w^2(M_1+M_2)^T(M_1+M_2)]^{-1} \cdot (VC)^T(S_1+S_2)^T(S_1+S_2)\mathbf{e}_\delta \tag{30}$$

The unbiased DFE settings can be obtained from equation (30) by scaling $\mathbf{f}$ by the reciprocal of the cursor, $\frac{1}{h(\delta)}$.

$$\mathbf{f_{opt}} = \frac{\mathbf{f}}{h(\delta)} \tag{31}$$

and the optimum feedback filter is,

$$\mathbf{b_{opt}} = \frac{[h(\delta+1)\ \cdots\ h(\delta+L_b)]^T}{h(\delta)} \tag{32}$$

6 Simulation Results

An asymmetric baud-spaced channel impulse response of

$$\mathbf{c1} = [0.183\ \ 0.916\ \ 0.289\ \ -0.183\ \ 0.092\ \ -0.046\ \ 0.018]$$

was considered. An unbiased MMSE-DFE with 4 feedforward taps and 3 feedback taps was used for both the normal-mode operation and the time-reversal operation of the DFE. The DFE taps were optimized to minimize MSE at the decision device input for each stream. The MSE performance curves for a conventional DFE, time-reversal DFE and the proposed diversity combining scheme (see Figure 1) are compared in Figure 2. Correct decision feedback was assumed for computing the MSE values. The optimal weighting factor for the diversity combining scheme was computed using equation (11) and the MSE using equation (9). The suboptimal DFE refers to the scenario where the DFE taps were optimized to minimize MSE at the decision device input for each stream. The diversity combining scheme yields a gain of about 1 dB for an MSE of 0.1 (i.e., -10 dB).

Furthermore, the symbol error rate(SER) curves for the conventional DFE, time-reversal DFE and the diversity combining scheme are shown in Figure 2 for the asymmetric channel $\mathbf{c_1}$. The effect of error propagation is incorporated in these curves, i.e., correct decision feedback was not assumed. In the diversity combining scheme with hard DFE a slicer was used as the decision device. On the other hand, in the diversity combining scheme with soft DFE a soft decision device (see [8]) was used. The transmitted symbols were assumed to be from a BPSK constellation. The SER plots show gains similar to those of the MSE plots.

In another experiment a symmetric baud-spaced channel with an impulse response of

$$\mathbf{c_2} = [0.2887\ \ 0.9129\ \ 0.2887]$$

was considered. A DFE with 4 feedforward taps and 3 feedback taps was used and $\mathbf{c_2}$ being a symmetric channel, the DFE settings were the same for both the normal-mode operation and the time-reversal operation of the DFE. Figure 3 compares the MSE performance curves with and without the diversity combining scheme. The suboptimal DFE refers to the scenario where the DFE taps were optimized to minimize MSE at the decision device input for each stream.

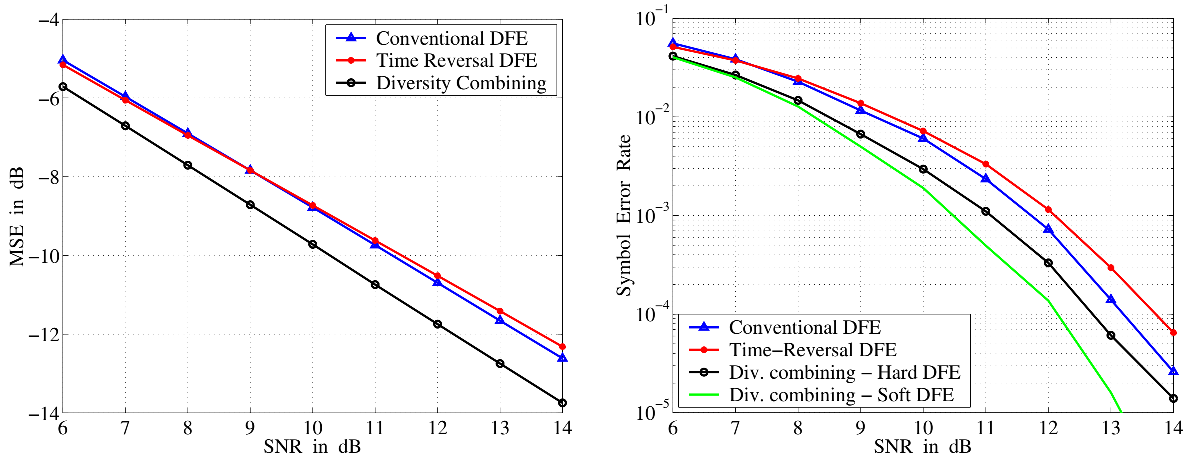

Figure 2: MSE and SER performance curves for an asymmetric channel

The optimal DFE, however, was optimized to minimize the MSE at the output of the diversity combining scheme and the DFE tap settings were computed using equations (31) and (32). Note that for the symmetric channel case the time-reversal mode operation will have the same performance as the normal-mode operation. Correct decision feedback was assumed while computing the MSE values.

The diversity combining scheme with the suboptimal DFE settings provides a gain of about 1 dB at an MSE of 0.1 over the normal-mode operation of the DFE. The diversity combining scheme with the optimal DFE tap settings provides an additional gain of 0.6 dB over the suboptimal case. However, in the presence of decision errors the same gains cannot be expected for the optimal DFE settings over the suboptimal DFE settings. This is due to the fact that the loss due to error propagation depends on the magnitude of the feedback filter coefficients and hence may be different for the two cases. Figure 3 also compares the SER plots for a conventional DFE, a DFE with ideal feedback and the diversity combining scheme with the suboptimal DFE tap settings. Gains similar to those of the MSE plots were observed.

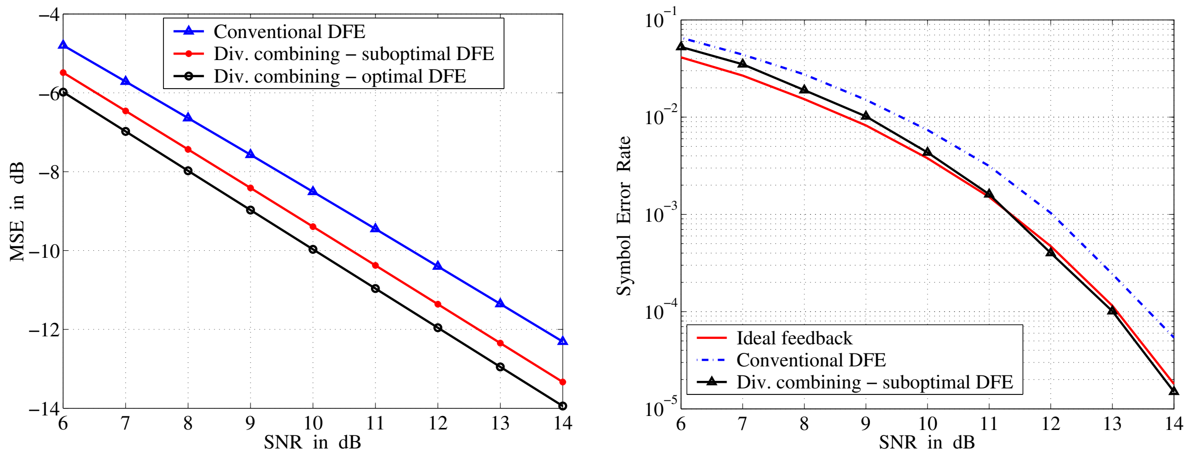

Figure 3: MSE and SER performance curves for a symmetric channel

7 Conclusions and Future Work

This paper considered the use of a time-reversal decision feedback equalizer (DFE) in conjunction with a conventional DFE. A receiver structure that combines the forward DFE outputs and the time-reversal DFE outputs to exploit the diversity between the two streams was proposed. An optimal combining scheme of the two streams that minimizes the mean-squared error (MSE) at the output of the receiver structure was derived. The inherent diversity between the soft outputs of the conventional DFE and a time-reversal DFE was quantified under a few simplifying assumptions. The DFE filter settings were optimized, for a channel with a symmetric impulse response, to minimize MSE at the output of the diversity combining block. Simulation results demonstrating performance gains for the proposed receiver structure were presented for both symmetric and asymmetric channels.

Current research focuses on the optimization of the DFE taps to minimize MSE at the output of the diversity combining block for an asymmetric channel. Furthermore, since the decisions at the output of the diversity combining block are more reliable than the tentative decisions in either of the normal-mode or the time-reversal DFE, it is possible to iteratively improve the performance by feeding back the more reliable decisions to the feedback filter of the DFE.

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